

Slides based on Haozhao Liang slides @ NSDII, Opatija 2012

Local covariant density functional constrained by the relativistic Hartree-Fock theory

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Outline

Introduction

Theoretical Framework

- Relativistic Hartree-Fock theory

- Zero-range reduction

- Fierz transformation

Results and Discussion

- Coupling strengths in different channels

- Dirac mass splitting

- Spin-isospin resonances

Summary and Perspectives

Work in progress

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Many-body systems and density functional theories

- ▶ Research on quantum mechanical many-body problems is essential in many areas of modern physics
 - ★ electrons in metal, atoms in molecule, electrons in atom, nucleons in nucleus ...
- ▶ Density functional theories (DFT) [Hohenberg & Kohn:1964](#)
 - ★ reducing the many-body problems formulated in terms of N-particle wave functions to the one-particle level with the local density distribution $\rho(\mathbf{r})$
 - ★ no other method achieves comparable accuracy at the same computational cost

Many-body systems and density functional theories

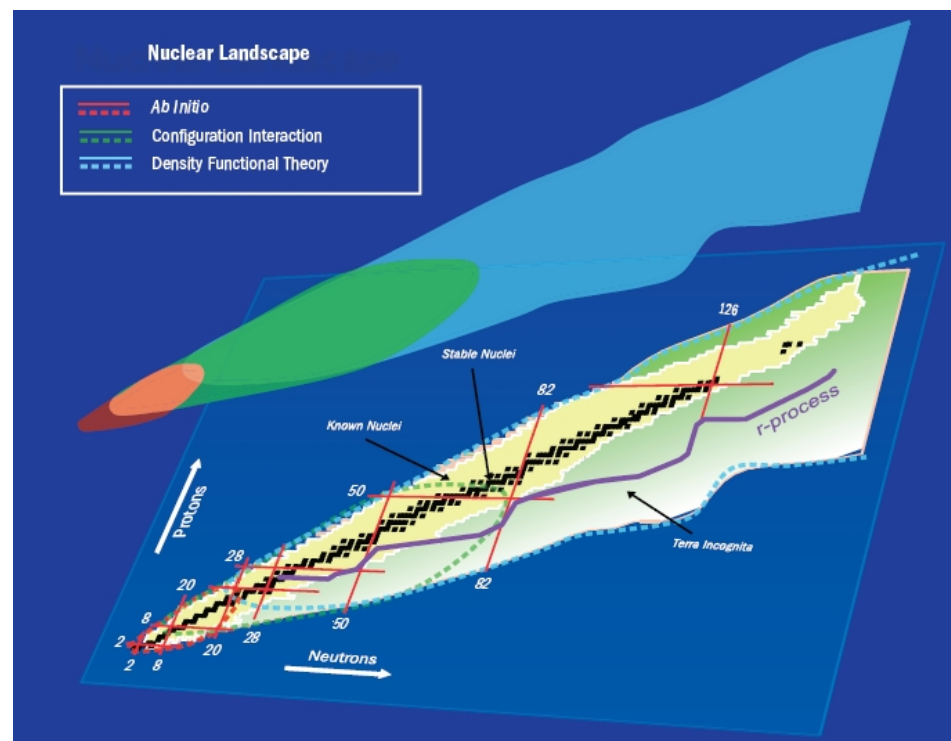
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- ▶ Density functional theories (DFT) [Hohenberg & Kohn:1964](#)
 - ★ reducing the many-body problems formulated in terms of N-particle wave functions to the one-particle level with the local density distribution $\rho(\mathbf{r})$
 - ★ no other method achieves comparable accuracy at the same computational cost
- ▶ Kohn-Sham scheme [Kohn & Sham:1965](#)
 - ★ for any interacting system, there exists a **local** single-particle (Kohn-Sham) potential $v_{\text{KS}}(\mathbf{r})$, such that the exact ground-state density of the interacting system can be reproduced by non-interacting particles moving in this local potential:
$$\rho(\mathbf{r}) = \rho_{\text{KS}}(\mathbf{r}) \equiv \sum_i^{\text{occ}} |\phi_i(\mathbf{r})|^2$$
 - ★ system energy density functional:

$$E = E[\rho(\mathbf{r})]$$

Nuclear DFT

- ▶ Nuclear DFT is a promising tool for investigating the ground-state and excited state properties of nuclei throughout the nuclear chart.
- ▶ Since the 1970s, lots of experience have been accumulated in implementing, adjusting, and using the DFT in nuclei.

Petkov&Stoitsov:1991, Bender:2003, Dobaczewski:INPC2010 plenary talk



Covariant density functional theory – RH theory

- ▶ The covariant version of DFT takes into account Lorentz symmetry.
 - ★ Causality is preserved and spin-orbit potential emerge naturally, ...
- ▶ CDFT in Hartree level (RH/RMF theory) has received wide attention due to its successful description of lots of nuclear phenomena.

[Serot:1986](#), [Ring:1996](#), [Vretenar:2005](#), [Meng:2006](#), [Paar:2007](#), [Nikšić:2011](#)

- ★ spin-orbit splittings
- ★ EoS in symmetric and asymmetric nuclear matter
- ★ ground-state properties of finite spherical and deformed nuclei
- ★ collective rotational and vibrational excitations
- ★ low-lying spectra of transitional nuclei involving quantum phase transitions
- ★

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Common problems in the isovector channels

- ▶ Difficult to disentangle the isovector-scalar (δ) and isovector-vector (ρ) channels, unless a tuning is performed based on selected microscopic calculations. [Roca-Maza:2011](#)
- ▶ Nuclear spin-isospin resonances, e.g., GTR and SDR, cannot be described without a direct fit to the data on their excitation energies.

Covariant density functional theory – RHF theory

- ▶ CDFT in Hartree-Fock level (RHF theory)

- ★ several attempts to include the Fock term in the relativistic framework

Bouyssy:1985,1987, Bernardos:1993, Marcos:2004

- ★ DDRHF theory achieved quantitative descriptions of binding energies and radii

Long, Giai, Meng, *PLB* **640**, 150 (2006); Long, Sagawa, Giai, Meng, *PRC* **76**, 034314 (2007);

Long, Sagawa, Meng, Giai, *EPL* **82**, 12001 (2008); Long, Ring, Giai, Meng, *PRC* **81**, 024308 (2010)

- ★ effective mass splitting in asymmetric nuclear matter can be described naturally

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- ★ nuclear spin-isospin resonances can be described in a fully self-consistent way

HL, Giai, Meng, *PRL* **101**, 122502 (2008); HL, Giai, Meng, *PRC* **79**, 064316 (2009);

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- ▶ RHF includes **non-local** potentials $v_{\text{HF}}(\mathbf{r}, \mathbf{r}')$, the simplicity of KS scheme is lost.
- ▶ RHF is much more complicated than RH theory.
- ▶ The computational cost is too expensive for including pairing, deformation, projection, cranking, ...

To construct RH functionals from RHF scheme

It is therefore highly desirable

- ▶ to stay within the conventional Kohn-Sham scheme in nuclear physics
- ▶ to find a covariant density functional based on only local potentials, yet keeping the merits of the exchange terms
- ▶ Possible/promising solution: construct RH functionals from RHF scheme
 - ★ take the constraints introduced by exchange terms of RHF scheme into account
- ▶ We start from an important observation: RHF functional PKO2 [[Long:2008](#)]
 - ★ well describes the neutron-proton Dirac mass splitting in asymmetric nuclear matter and nuclear spin-isospin resonances
 - ★ only includes σ -, ω -, ρ -mesons, but not π -meson
 - ★ masses of mesons are heavy $\Rightarrow$ zero-range approximation is reasonable
 - ★ Fierz transformation: Fock terms $\Rightarrow$ local Hartree terms

In the present work

- ▶ To construct RH functional from RHF scheme by the following procedure
 - ★ start with RHF parametrization PKO2
 - ★ perform the zero-range reduction
 - ★ perform the Fierz transformation
- ▶ With such RH functional thus obtained, to investigate
 - ★ proton-neutron Dirac mass splitting in neutron matter
 - ★ Gamow-Teller and spin-dipole resonances

Goal(s)

- ▶ To verify whether the important effects of exchange terms can be maintained by the mapping from RHF functional to RH functional.

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Covariant density functional theory – RHF theory

- Effective Lagrangian density [Bouyssy:1987](#), [Long:2006](#)

$$\mathcal{L} = \bar{\psi} \left[i\gamma^\mu \partial_\mu - M - \mathbf{g}_\sigma \sigma - \gamma^\mu \left(\mathbf{g}_\omega \omega_\mu + \mathbf{g}_\rho \vec{\tau} \cdot \vec{\rho}_\mu + e \frac{1 - \tau_3}{2} A_\mu \right) \right] \psi \\ + \frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{4} \Omega^{\mu\nu} \Omega_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} \vec{R}_{\mu\nu} \cdot \vec{R}^{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{\rho}^\mu \cdot \vec{\rho}_\mu - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \quad (1)$$

- System Hamiltonian

$$\mathcal{H} = \mathcal{T}^{00} = \frac{\partial \mathcal{L}}{\partial \dot{\phi}_i} \dot{\phi}_i - \mathcal{L} \quad (2)$$

- Ground-state trial wave function

$$|\Phi_0\rangle = \prod_a c_a^\dagger |0\rangle \quad (3)$$

- Energy functional of the system

$$E = \langle \Phi_0 | H | \Phi_0 \rangle = E_k + E_\sigma^D + E_\omega^D + E_\rho^D + E_A^D + E_\sigma^E + E_\omega^E + E_\rho^E + E_A^E \quad (4)$$

Zero-range reduction

- ▶ Yukawa propagators of the mesons

$$D_i(\mathbf{r}, \mathbf{r}') = \frac{1}{4\pi} \frac{e^{-m_i|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r} - \mathbf{r}'|}, \quad D_i(\mathbf{q}) = \frac{1}{m_i^2 + \mathbf{q}^2} \quad (5)$$

for $m_i \gg q$,

$$D_i(\mathbf{q}) \approx \frac{1}{m_i^2} - \frac{\mathbf{q}^2}{m_i^4} + \dots \quad \Rightarrow \quad D_i(\mathbf{r}, \mathbf{r}') \approx \frac{1}{m_i^2} \delta(\mathbf{r} - \mathbf{r}') \quad (6)$$

within the zero-order approximation.

- ▶ Zero-range reduction of meson-nucleon couplings

$$\alpha_S^{\text{HF}} = -\frac{g_\sigma^2}{m_\sigma^2}, \quad \alpha_V^{\text{HF}} = \frac{g_\omega^2}{m_\omega^2}, \quad \alpha_{tV}^{\text{HF}} = \frac{g_\rho^2}{m_\rho^2}, \quad (7)$$

Fierz transformation (I)

- Sixteen Dirac matrices form a complete system

$$O^S = 1, O^V = \gamma^\mu, O^T = \sigma^{\mu\nu}, O^{PS} = \gamma^5, O^{PV} = \gamma^5 \gamma^\mu$$

so that any one can be expressed as a linear superposition of variants with a changed sequence of spinors,

$$(\bar{a} O^i d)(\bar{c} O_j b) = \sum_k c_{ik} (\bar{a} O^k b)(\bar{c} O_k d), \quad (8)$$

with the coefficients c_{ik} in the so-called Fierz table [Fierz:1937](#), [Okun:1982](#), [Sulaksono:2003](#)

	S	V	T	PS	PV
S	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{4}$	$-\frac{1}{4}$
V	1	$-\frac{1}{2}$	0	-1	$-\frac{1}{2}$
T	3	0	$-\frac{1}{2}$	3	0
PS	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{4}$
PV	-1	$-\frac{1}{2}$	0	1	$-\frac{1}{2}$

(9)

- For the isospin coefficients,

$$\delta_{q_a q_d} \delta_{q_c q_b} = \frac{1}{2} \left[\delta_{q_a q_b} \delta_{q_c q_d} + \langle q_a | \vec{\tau} | q_b \rangle \cdot \langle q_c | \vec{\tau} | q_d \rangle \right], \quad (10a)$$

$$\langle q_a | \vec{\tau} | q_d \rangle \cdot \langle q_c | \vec{\tau} | q_b \rangle = \frac{1}{2} \left[3 \delta_{q_a q_b} \delta_{q_c q_d} - \langle q_a | \vec{\tau} | q_b \rangle \cdot \langle q_c | \vec{\tau} | q_d \rangle \right]. \quad (10b)$$

Fierz transformation (II)

► Fierz transformation: from α^{HF} to α^{H}

$$\alpha_S^{\text{H}} = +\frac{7}{8}\alpha_S^{\text{HF}} - \frac{4}{8}\alpha_V^{\text{HF}} - \frac{12}{8}\alpha_{tV}^{\text{HF}} \quad (11a)$$

$$\alpha_{tS}^{\text{H}} = -\frac{1}{8}\alpha_S^{\text{HF}} - \frac{4}{8}\alpha_V^{\text{HF}} + \frac{4}{8}\alpha_{tV}^{\text{HF}} \quad (11b)$$

$$\alpha_V^{\text{H}} = -\frac{1}{8}\alpha_S^{\text{HF}} + \frac{10}{8}\alpha_V^{\text{HF}} + \frac{6}{8}\alpha_{tV}^{\text{HF}} \quad (11c)$$

$$\alpha_{tV}^{\text{H}} = -\frac{1}{8}\alpha_S^{\text{HF}} + \frac{2}{8}\alpha_V^{\text{HF}} + \frac{6}{8}\alpha_{tV}^{\text{HF}} \quad (11d)$$

$$\alpha_T^{\text{H}} = -\frac{1}{16}\alpha_S^{\text{HF}} \quad (11e)$$

$$\alpha_{tT}^{\text{H}} = -\frac{1}{16}\alpha_S^{\text{HF}} \quad (11f)$$

$$\alpha_{PS}^{\text{H}} = -\frac{1}{8}\alpha_S^{\text{HF}} + \frac{4}{8}\alpha_V^{\text{HF}} + \frac{12}{8}\alpha_{tV}^{\text{HF}} \quad (11g)$$

$$\alpha_{tPS}^{\text{H}} = -\frac{1}{8}\alpha_S^{\text{HF}} + \frac{4}{8}\alpha_V^{\text{HF}} - \frac{4}{8}\alpha_{tV}^{\text{HF}} \quad (11h)$$

$$\alpha_{PV}^{\text{H}} = +\frac{1}{8}\alpha_S^{\text{HF}} + \frac{2}{8}\alpha_V^{\text{HF}} + \frac{6}{8}\alpha_{tV}^{\text{HF}} \quad (11i)$$

$$\alpha_{tPV}^{\text{H}} = +\frac{1}{8}\alpha_S^{\text{HF}} + \frac{2}{8}\alpha_V^{\text{HF}} - \frac{2}{8}\alpha_{tV}^{\text{HF}} \quad (11j)$$

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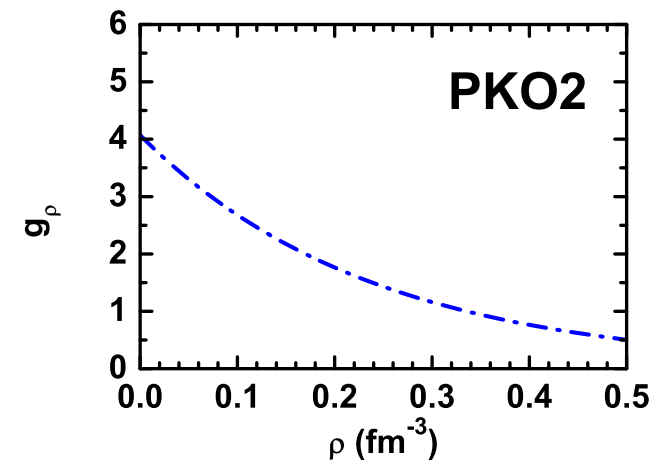
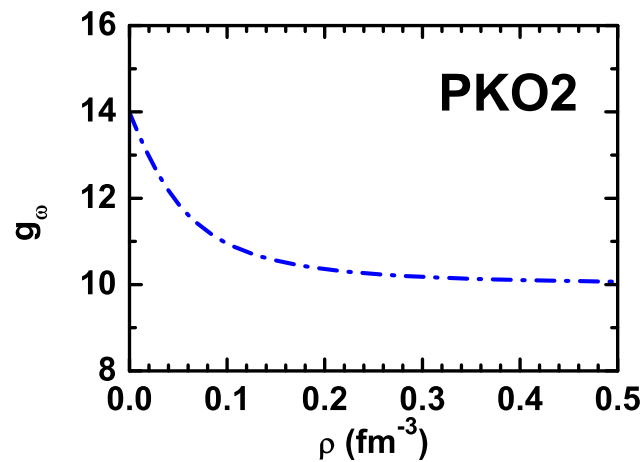
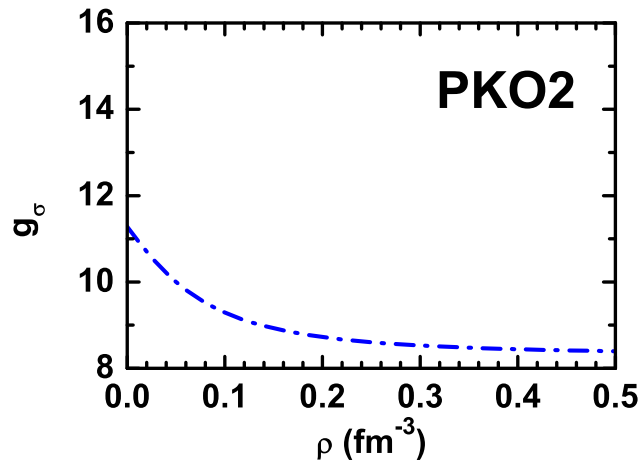
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Work in progress

Nucleon-meson coupling strengths of PKO2

- ▶ Starting point: nucleon-meson coupling strengths g_σ , g_ω , and g_ρ of PKO2



Long, Sagawa, Meng, Giai, *EPL* **82**, 12001 (2008)

- ▶ Zero-range reduction

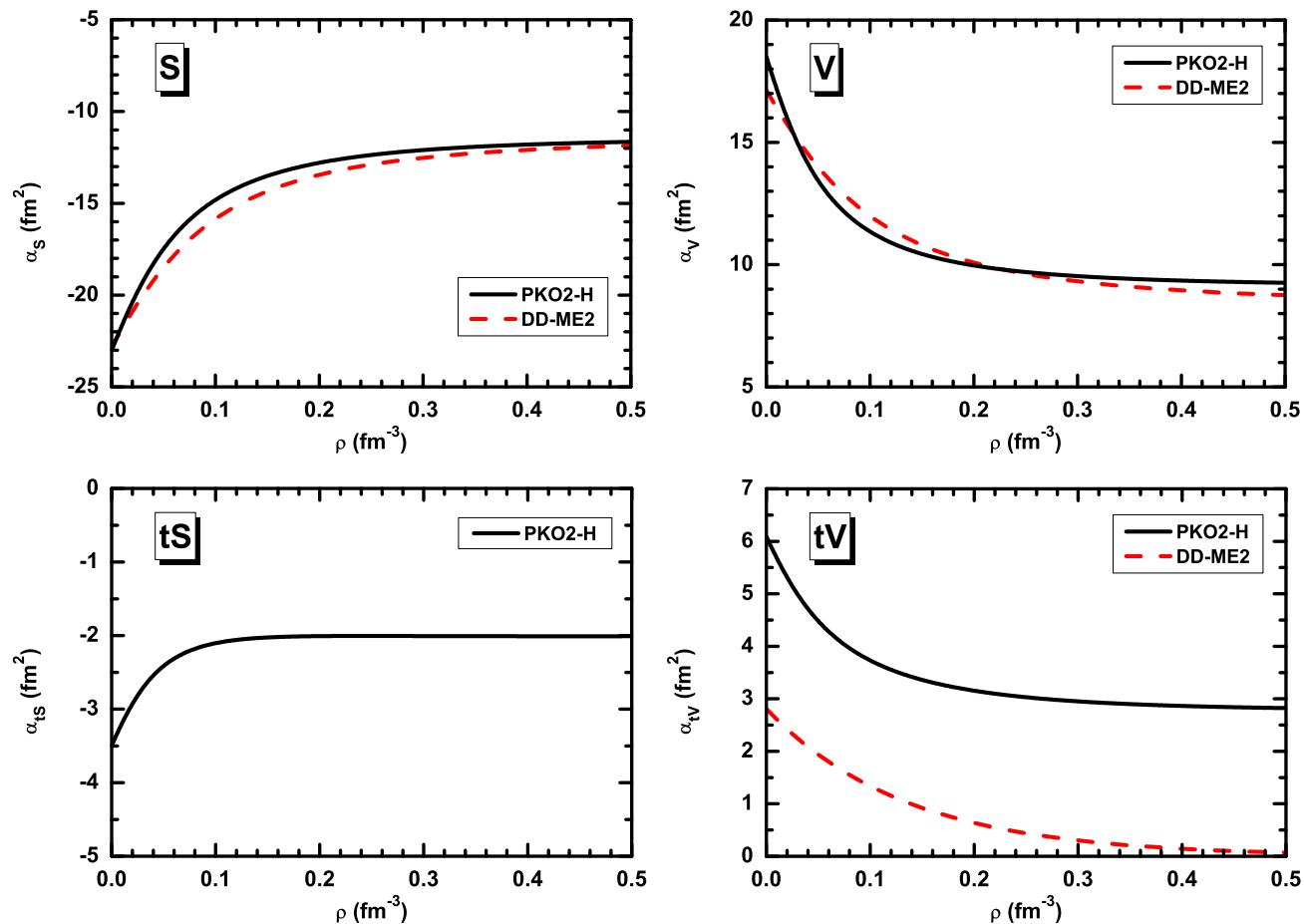
$$\alpha_S^{\text{HF}} = -\frac{g_\sigma^2}{m_\sigma^2}, \quad \alpha_V^{\text{HF}} = \frac{g_\omega^2}{m_\omega^2}, \quad \alpha_{tV}^{\text{HF}} = \frac{g_\rho^2}{m_\rho^2},$$

- ▶ Fierz transformation

$$\alpha_j^{\text{H}} = \sum_k c_{jk} \alpha_k^{\text{HF}},$$

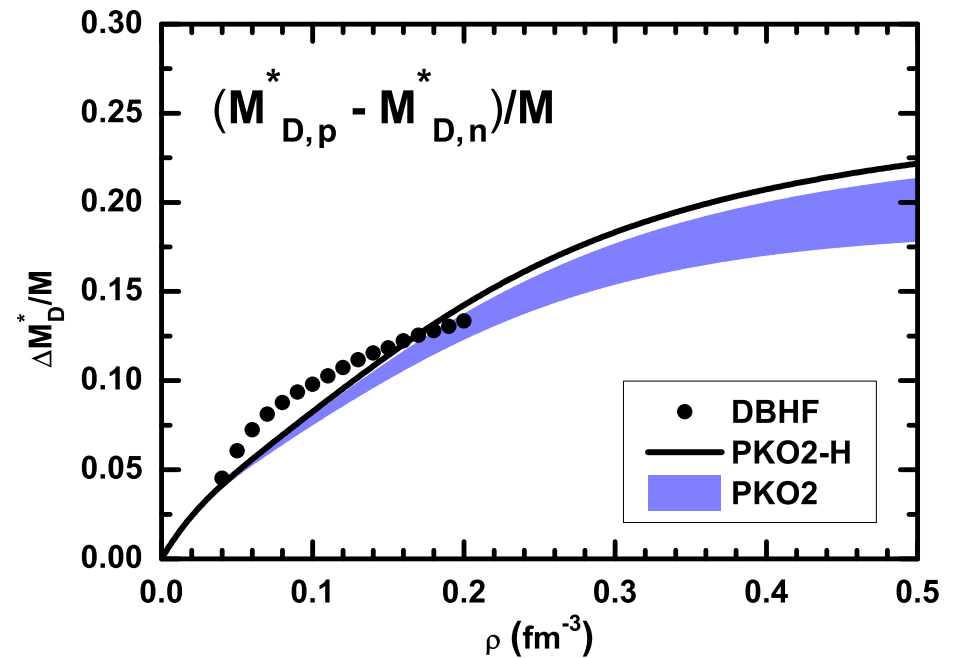
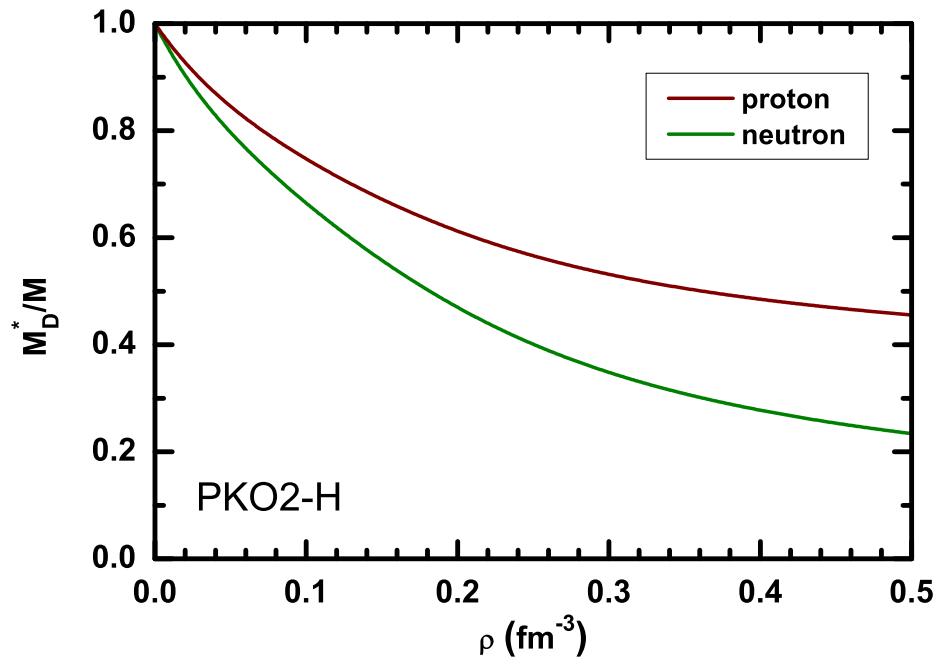
RHF equivalent zero-range coupling strengths (I)

- ▶ The RHF equivalent parametrization derived from PKO2 is called “**PKO2-H**”.



- ▶ α_S^H and α_V^H are consistent with those of DD-ME2 [Lalazisis:2005](#)
- ▶ α_{tS}^H appears $\Rightarrow$ proton-neutron Dirac mass splitting in asymmetric nuclear matter
- ▶ α_{tV}^H is modified for another delicate balance between tS and tV channels

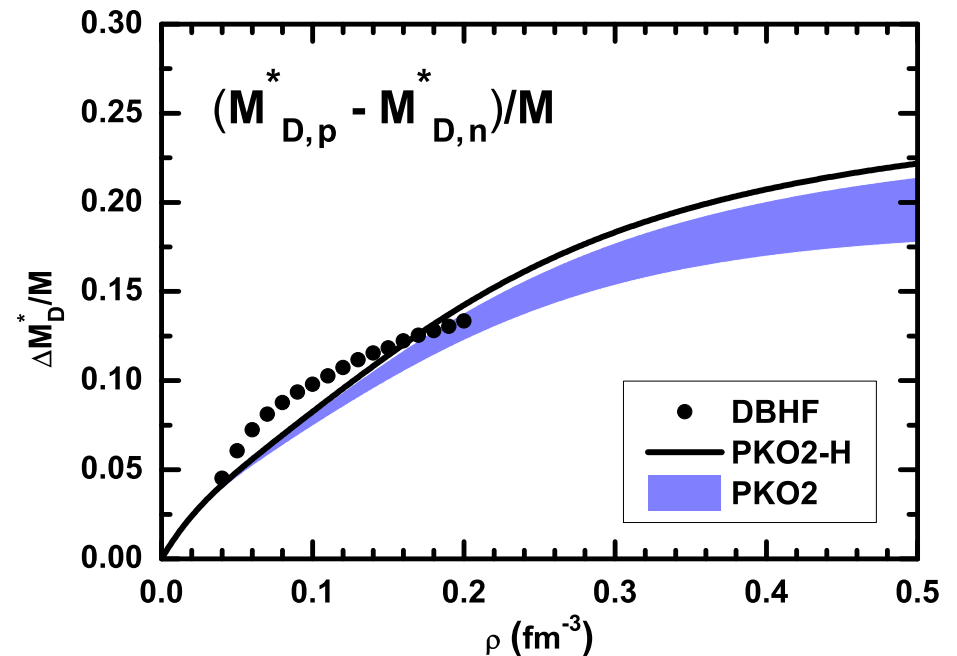
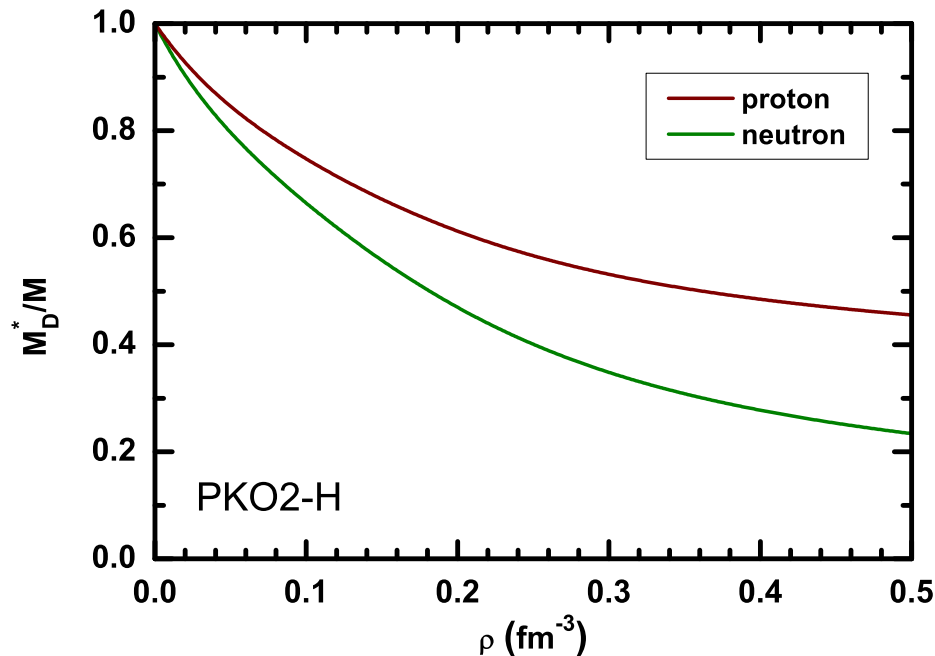
Dirac mass and its isospin splitting



DBHF: van Dalen, *et al.*, *EPJA* **31**, 29 (2007)

- It is found that $M_{D,p}^* > M_{D,n}^*$.
- The splitting behavior in neutron matter is quantitatively consistent with the prediction of the Dirac-Brueckner-Hartree-Fock (DBHF) calculations.

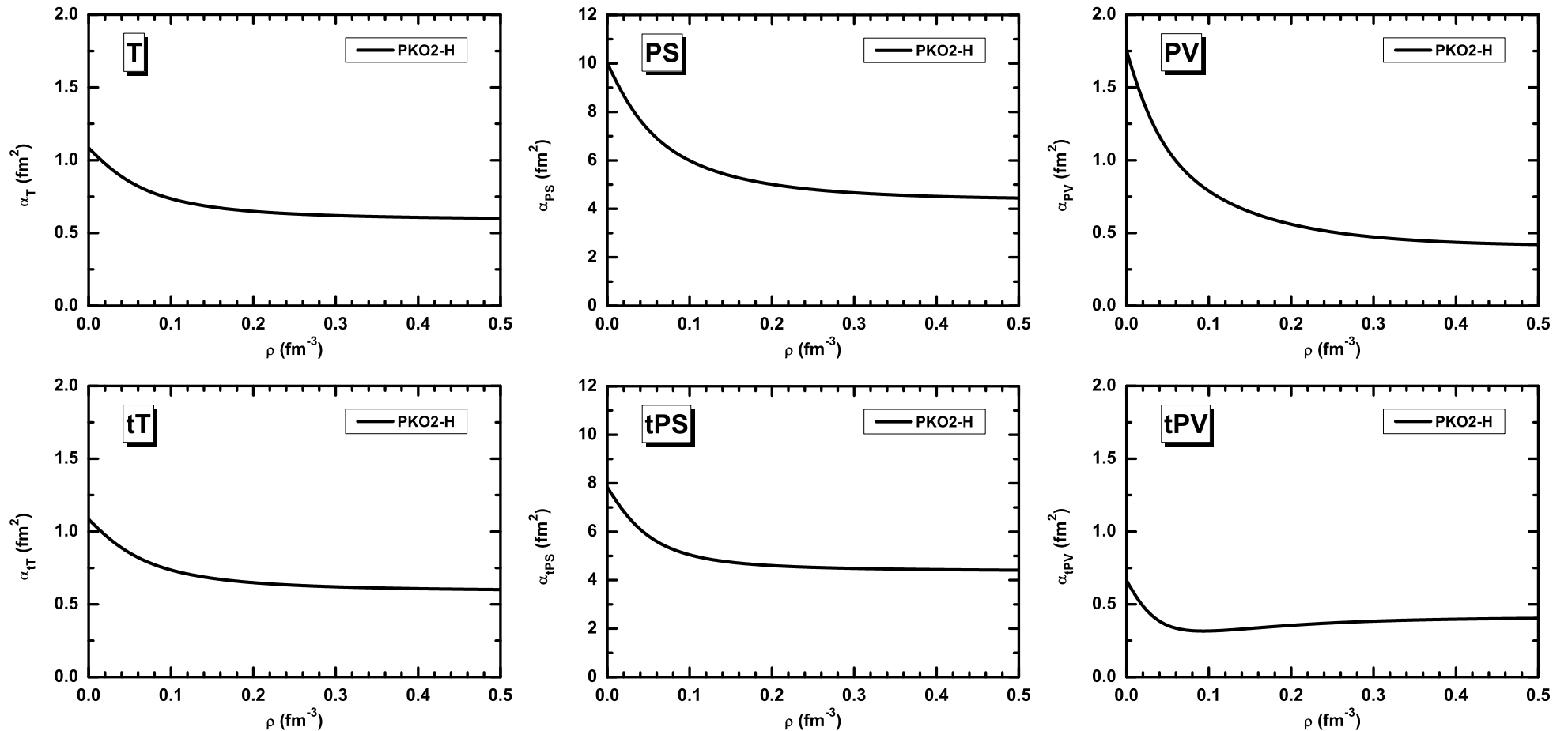
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- ▶ The splitting behavior in neutron matter is quantitatively consistent with the prediction of the Dirac-Brueckner-Hartree-Fock (DBHF) calculations.
- ▶ The constraints introduced by the Fock terms of the RHF scheme into the tS channel of the present density functional are straight forward and quite robust.

RHF equivalent zero-range coupling strengths (II)



- ▶ $\alpha_{(t)T}^H$, $\alpha_{(t)PS}^H$, and $\alpha_{(t)PV}^H$ are explicitly determined by exchange effects of RHF scheme
- ▶ $(t)PS$ and $(t)PV$ channels vanish in ground-state descriptions due to the parity conservation, but crucial for spin-isospin resonances

Particle-hole residual interactions in charge-exchange channel

- ▶ Particle-hole (*ph*) residual interactions

- ▶ *tS* channel: $V_{tS}(1, 2) = \alpha_{tS}^H [\gamma_0 \vec{\tau}]_1 \cdot [\gamma_0 \vec{\tau}]_2 \delta(\mathbf{r}_1 - \mathbf{r}_2),$ (12a)

- ▶ *tV* channel: $V_{tV}(1, 2) = \alpha_{tV}^H [\gamma_0 \gamma^\mu \vec{\tau}]_1 \cdot [\gamma_0 \gamma_\mu \vec{\tau}]_2 \delta(\mathbf{r}_1 - \mathbf{r}_2),$ (12b)

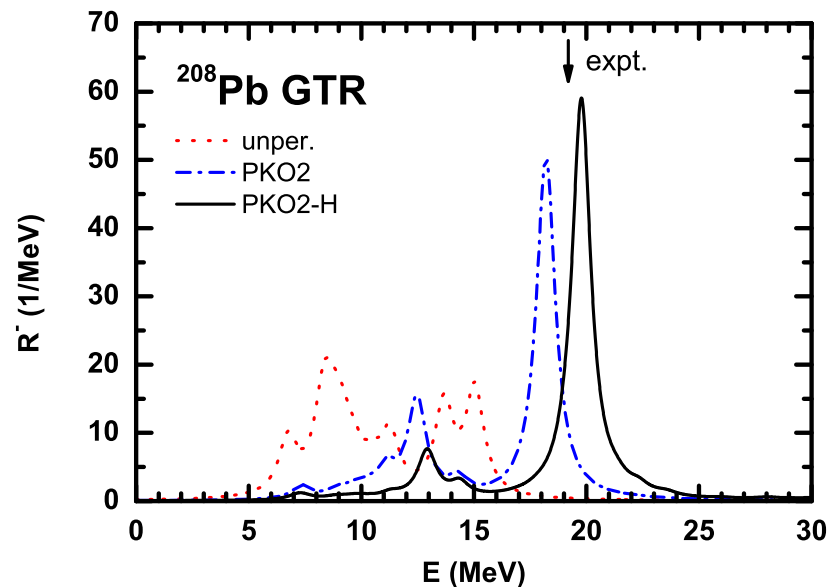
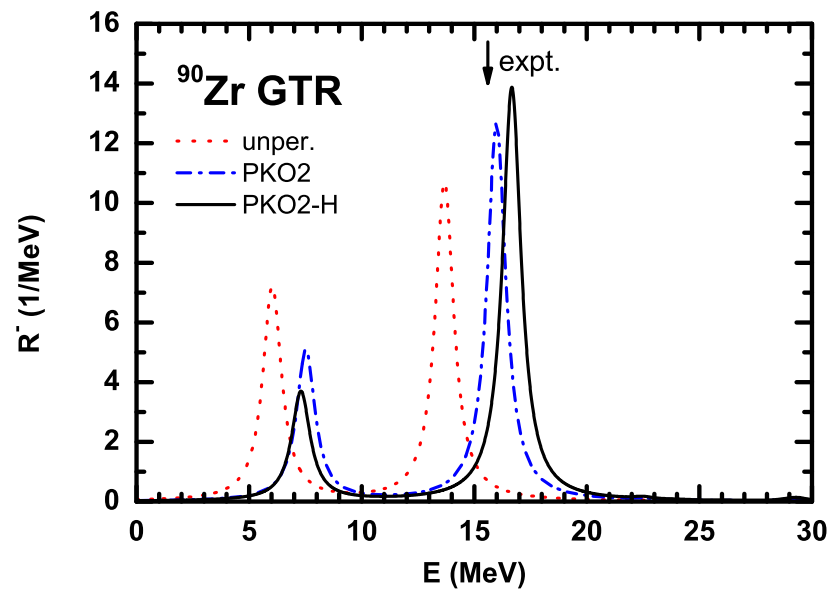
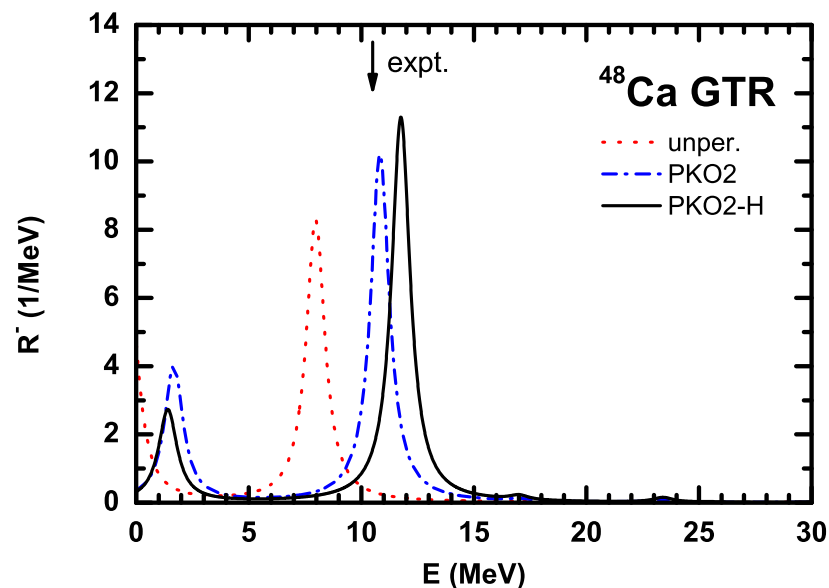
- ▶ *tT* channel: $V_{tT}(1, 2) = \alpha_{tT}^H [\gamma_0 \sigma^{\mu\nu} \vec{\tau}]_1 \cdot [\gamma_0 \sigma_{\mu\nu} \vec{\tau}]_2 \delta(\mathbf{r}_1 - \mathbf{r}_2),$ (12c)

- ▶ *tPS* channel: $V_{tPS}(1, 2) = \alpha_{tPS}^H [\gamma_0 \gamma_5 \vec{\tau}]_1 \cdot [\gamma_0 \gamma_5 \vec{\tau}]_2 \delta(\mathbf{r}_1 - \mathbf{r}_2),$ (12d)

- ▶ *tPV* channel: $V_{tPV}(1, 2) = \alpha_{tPV}^H [\gamma_0 \gamma_5 \gamma^\mu \vec{\tau}]_1 \cdot [\gamma_0 \gamma_5 \gamma_\mu \vec{\tau}]_2 \delta(\mathbf{r}_1 - \mathbf{r}_2).$ (12e)

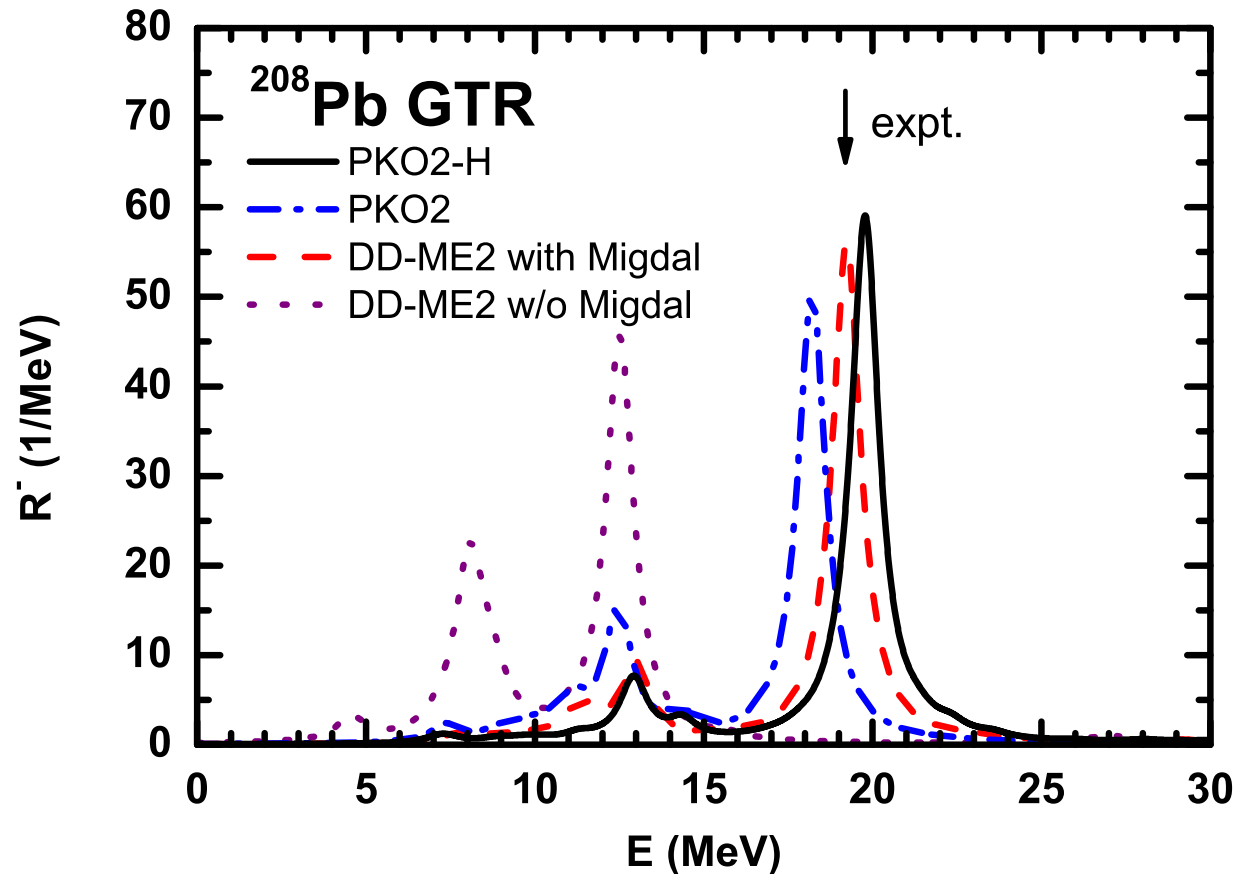
- ▶ For the charge-exchange spin-flip modes, the *tT* and *tPV* channels are expected to play the dominant roles, where the operator $[\boldsymbol{\sigma} \vec{\tau}] \cdot [\boldsymbol{\sigma} \vec{\tau}]$ is sandwiched by the large components of wave functions.

Gamow-Teller resonances



- ▶ GTR excitation energies can be well reproduced by the ph residual interactions of PKO2-H.
- ▶ The present results are similar as those by the original RHF+RPA.
- ▶ The difference is due to the zero-range approximation.

Gamow-Teller resonances in ^{208}Pb

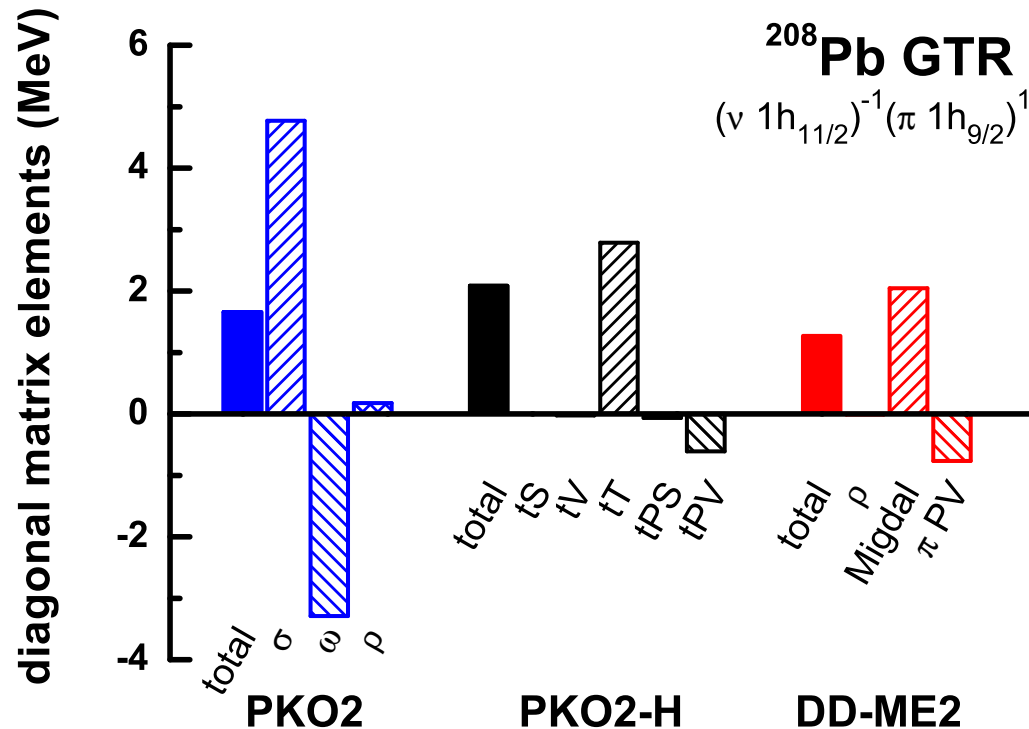


- It is fitted for DD-ME2 by adjusting the Migdal term.

RHF+RPA: HL, Giai, Meng, *PRL* **101**, 122502 (2008)

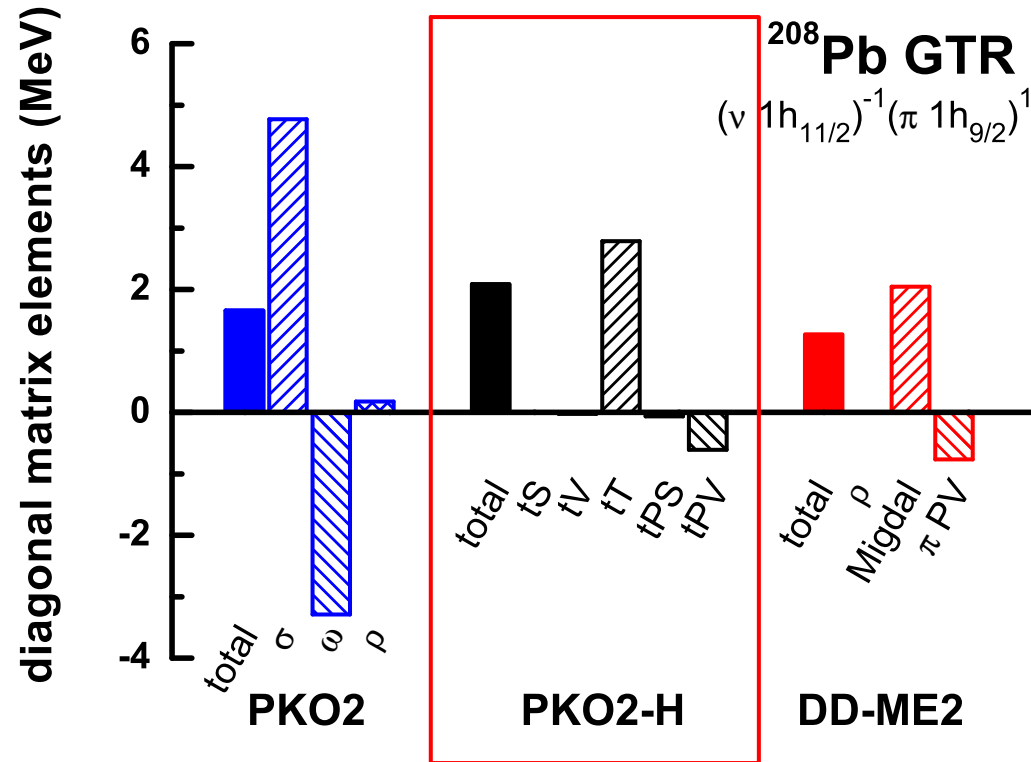
RH+RPA: Paar, Nikšić, Vretenar, Ring, *PRC* **69**, 054303 (2004)

Physical mechanisms of GTR



- ▶ In RHF+RPA (PKO2), σ - and ω -mesons play the most important role in determining the properties of GTR via the exchange terms.
- ▶ In the RHF equivalent RPA (PKO2-H), tT and tPV channels are most important, their coupling strengths are intrinsically determined by Fierz transformation.
- ▶ In conventional RH+RPA (DD-ME2), the free π residual interaction is attractive and the Migdal term is repulsive by fitting to experimental data.

Physical mechanisms of GTR by PKO2-H

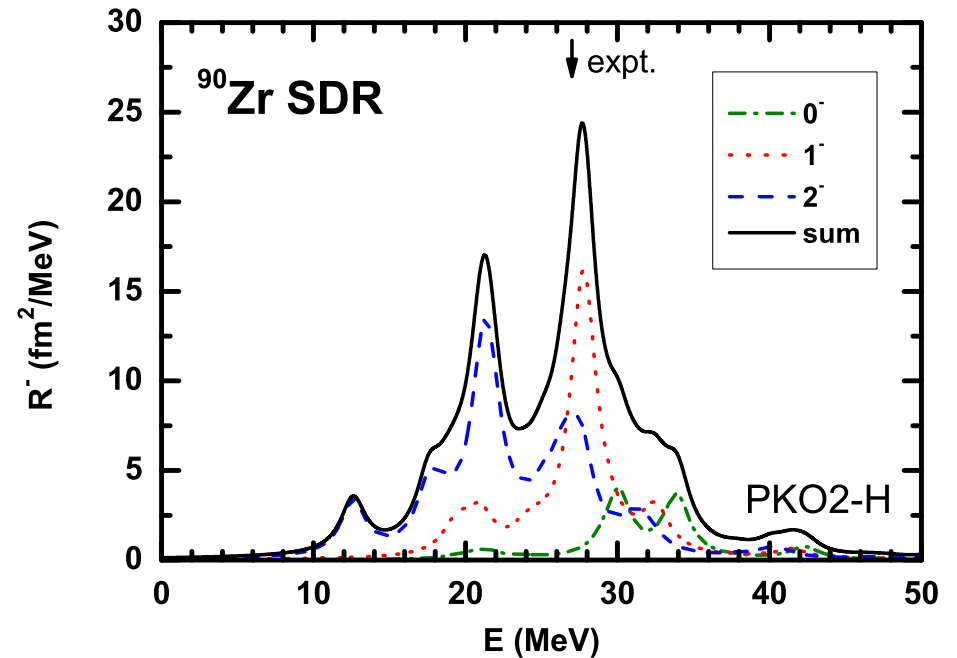
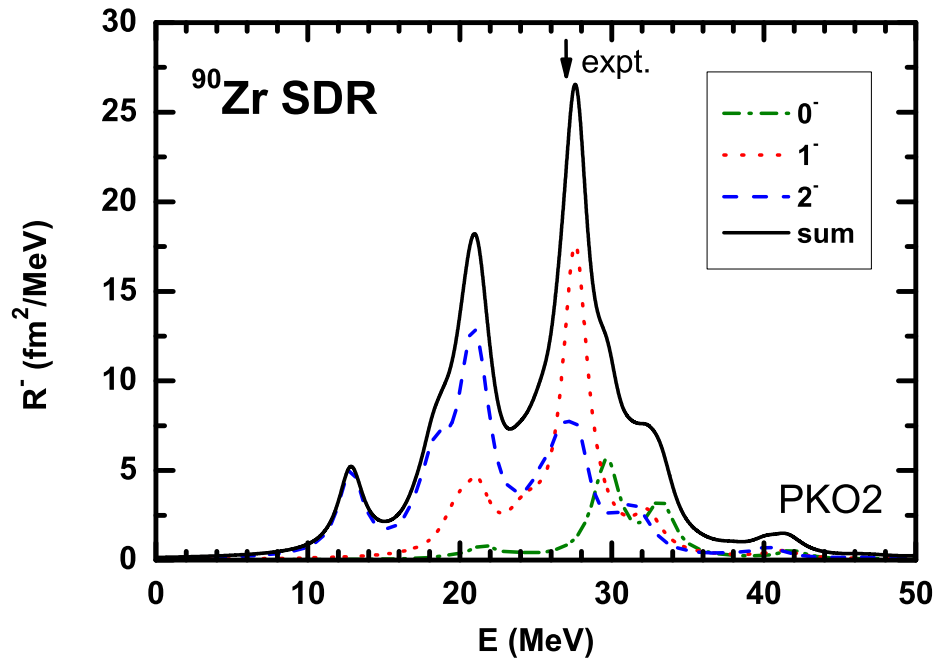


- ▶ The dominant ph interactions are the operator $[\boldsymbol{\sigma}\vec{\tau}] \cdot [\boldsymbol{\sigma}\vec{\tau}]$ sandwiched by large components of w.f., i.e., $[\sigma^{ij}\vec{\tau}] \cdot [\sigma_{ij}\vec{\tau}] \propto 2\alpha_{tT}^H$ and $[\gamma_5\gamma^i\vec{\tau}] \cdot [\gamma_5\gamma_i\vec{\tau}] \propto -\alpha_{tPV}^H$
- ▶ The net contribution is then proportional to

$$2\alpha_{tT}^H - \alpha_{tPV}^H = -\frac{1}{3} (\alpha_S^H + \alpha_V^H)$$

- ▶ Total ph strengths are determined by the delicate balance between S and V channels.

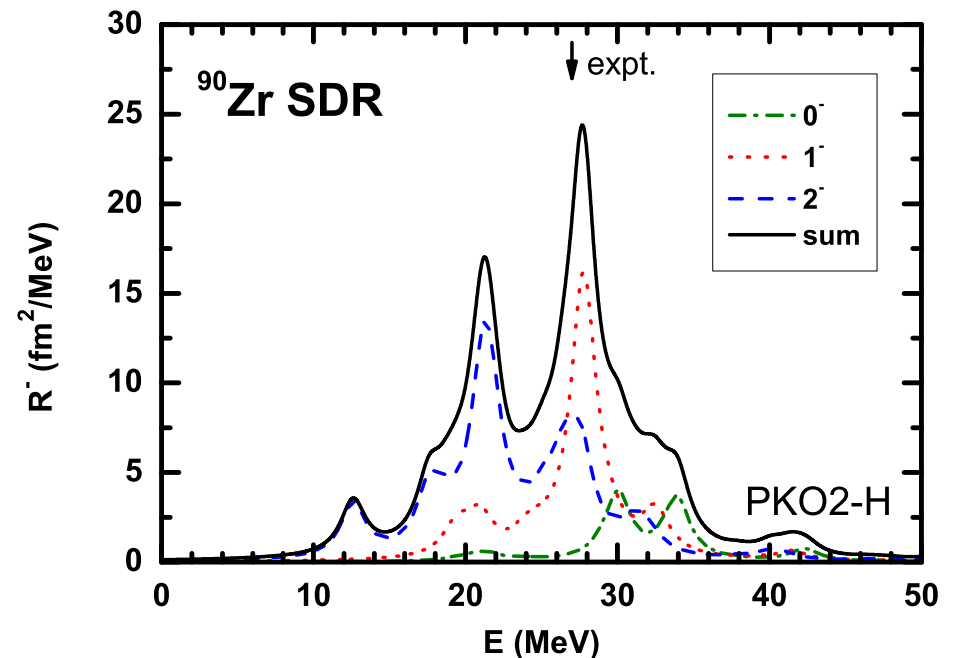
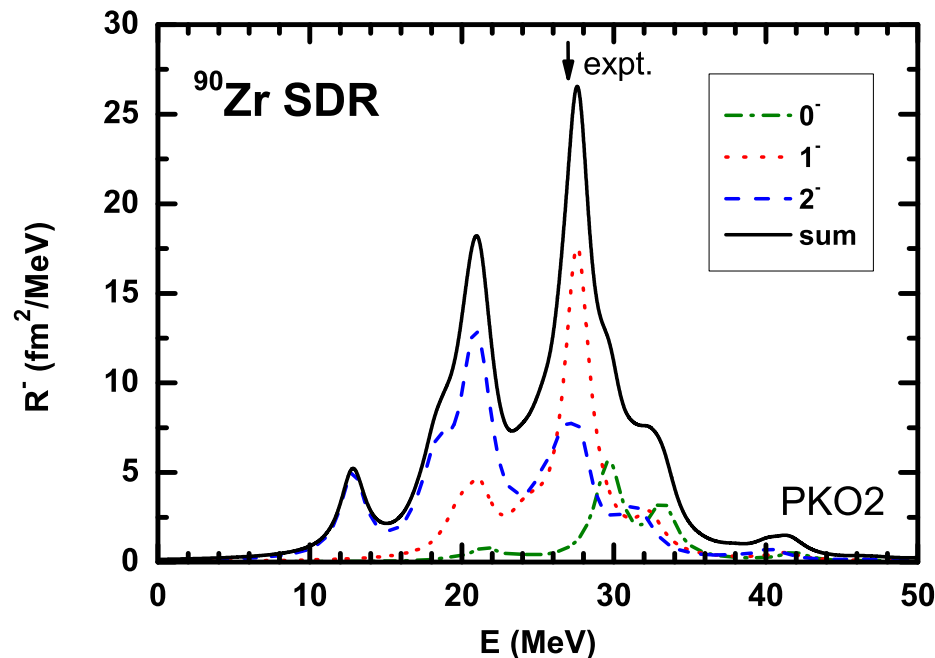
Spin-dipole resonances



expt: Yako, Sagawa, Sakai, *PRC* **74**, 051303 (2006)

- ▶ Not only the total strengths but also the individual contribution from different spin-parity J^π components are almost identical.
- ▶ The energy hierarchy $E(2^-) < E(1^-) < E(0^-)$ can be obtained naturally in the present RPA calculations in the local scheme.

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- ▶ The energy hierarchy $E(2^-) < E(1^-) < E(0^-)$ can be obtained naturally in the present RPA calculations in the local scheme.
- ▶ The constraints introduced by the Fock terms of the RHF scheme into the ph residual interactions are also straight forward and quite robust.

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- Relativistic Hartree-Fock theory
- Zero-range reduction
- Fierz transformation

Results and Discussion

- Coupling strengths in different channels
- Dirac mass splitting
- Spin-isospin resonances

Summary and Perspectives

Work in progress

Summary and Perspectives

Summary

- ★ A new method is proposed to take into account the Fock terms in local covariant density functionals.
- ✓ The advantages of existing RH functionals can be maintained, while the problems in the isovector channel can be solved.
 - ★ The neutron-proton Dirac mass splitting in asymmetric nuclear matter is in a very good agreement with the prediction of DBHF.
 - ★ The properties of GTR and SDR can be reproduced in a natural way.

Perspectives

- This opens a new door for the development of nuclear local covariant density functionals with proper isoscalar and isovector properties in the future.

Outline

Introduction

Theoretical Framework

- Relativistic Hartree-Fock theory
- Zero-range reduction
- Fierz transformation

Results and Discussion

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Summary and Perspectives

Work in progress

Work in progress - I

Project Build a local covariant energy density functional including all terms in the Lagrangian allowed by the symmetries but only considering as free parameters the ones corresponding to the S, V and tV channels, the rest of the channels will be determined by the Fierz transformations within the same Hartree scheme in a fully self-consistent manner.

Work in progress - II

Fierz transformations

$$\alpha^H(S, V, tS, tV, T, tT, PS, tPS, PV, tPV) \rightarrow \alpha^H(S, V, tV)$$

$$\alpha_{tS}^H(\rho) = -\frac{1}{15}\alpha_S^H(\rho) - \frac{2}{3}\alpha_V^H(\rho) + \frac{6}{5}\alpha_{tV}^H(\rho),$$

$$\alpha_T^H(\rho) = -\frac{1}{10}\alpha_S^H(\rho) - \frac{1}{5}\alpha_{tV}^H(\rho),$$

$$\alpha_{tT}^H(\rho) = -\frac{1}{10}\alpha_S^H(\rho) - \frac{1}{5}\alpha_{tV}^H(\rho),$$

$$\alpha_{PS}^H(\rho) = \frac{1}{5}\alpha_S^H(\rho) + \frac{12}{5}\alpha_{tV}^H(\rho),$$

$$\alpha_{tPS}^H(\rho) = -\frac{1}{3}\alpha_S^H(\rho) + \frac{2}{3}\alpha_V^H(\rho) - 2\alpha_{tV}^H(\rho),$$

$$\alpha_{PV}^H(\rho) = \frac{2}{5}\alpha_S^H(\rho) + \frac{9}{5}\alpha_{tV}^H(\rho),$$

$$\alpha_{tPV}^H(\rho) = \frac{2}{15}\alpha_S^H(\rho) + \frac{1}{3}\alpha_V^H(\rho) - \frac{2}{5}\alpha_{tV}^H(\rho).$$

The reduction of the base of operators with which you represent your problem determines the couplings of the others. This does not mean that the mapping is *perfect* but we have checked that it approximately works for the HF $\rightarrow$ H case with PKO2.

Work in progress - III

Lagrangian

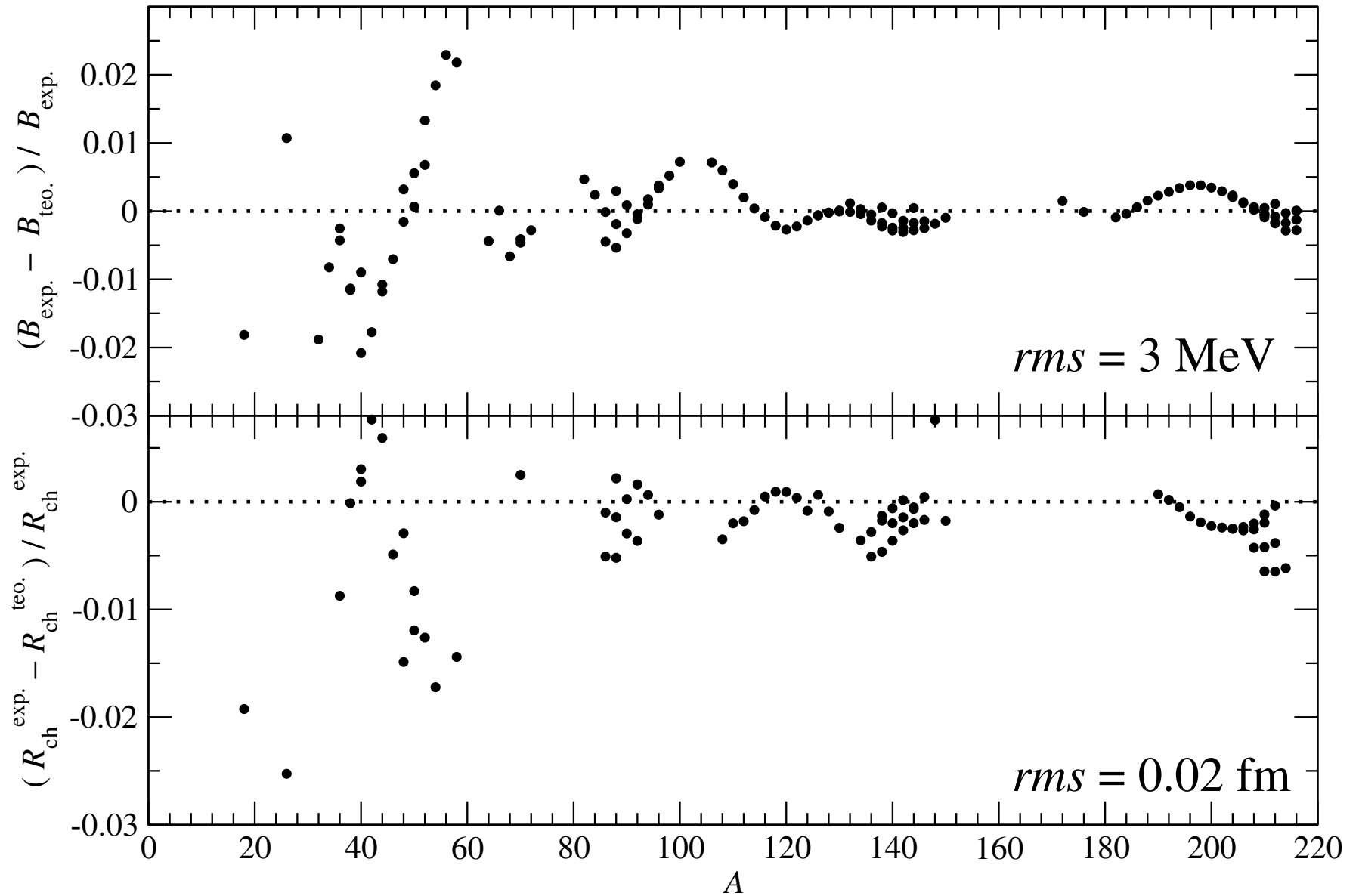
$$\begin{aligned}\mathcal{L} = & \bar{\psi}(i\gamma^\mu\partial_\mu - M)\psi \\ & - \frac{1}{2}\alpha_S(\bar{\psi}\psi)(\bar{\psi}\psi) - \frac{1}{2}\delta_S(\partial_\nu\bar{\psi}\psi)(\partial^\nu\bar{\psi}\psi) - \frac{1}{2}\alpha_{tS}(\bar{\psi}\vec{\tau}\psi) \cdot (\bar{\psi}\vec{\tau}\psi) \\ & - \frac{1}{2}\alpha_V(\bar{\psi}\gamma^\mu\psi)(\bar{\psi}\gamma_\mu\psi) - \frac{1}{2}\alpha_{tV}(\bar{\psi}\vec{\tau}\gamma^\mu\psi) \cdot (\bar{\psi}\vec{\tau}\gamma_\mu\psi) \\ & - \frac{1}{2}\alpha_T(\bar{\psi}\sigma^{\mu\nu}\psi)(\bar{\psi}\sigma_{\mu\nu}\psi) - \frac{1}{2}\alpha_{tT}(\bar{\psi}\sigma^{\mu\nu}\vec{\tau}\psi) \cdot (\bar{\psi}\sigma_{\mu\nu}\vec{\tau}\psi) \\ & - \frac{1}{2}\alpha_{PS}(\bar{\psi}\gamma^5\psi)(\bar{\psi}\gamma^5\psi) - \frac{1}{2}\alpha_{tPS}(\bar{\psi}\gamma^5\vec{\tau}\psi) \cdot (\bar{\psi}\gamma^5\vec{\tau}\psi) \\ & - \frac{1}{2}\alpha_{PV}(\bar{\psi}\gamma^5\gamma^\mu\psi)(\bar{\psi}\gamma^5\gamma_\mu\psi) - \frac{1}{2}\alpha_{tPV}(\bar{\psi}\gamma^5\gamma^\mu\vec{\tau}\psi) \cdot (\bar{\psi}\gamma^5\gamma_\mu\vec{\tau}\psi) \\ & - e\bar{\psi}\gamma^\mu A_\mu \frac{(1-\tau_3)}{2}\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu},\end{aligned}$$

Work in progress - IV

Test

The aim is twofold. On one side, we want to test the **feasibility of optimizing a localized Relativistic Hartree** functional with density dependent couplings where the scalar-isovector channel is completely determined from the other spin-isospin channels via the Fierz transformations. Note that for the test presented here, the six remaining channels —i.e. isoscalar- and isovector- tensor, pseudo-vector and pseudo-scalar— are completely neglected. And, on the other side, we want to **find a good starting point before fitting the complete localized functional**. For testing purposes we use the same fitting procedure used to fit DD-ME δ and the DD-PC1 ansatz for the density dependence of the coupling constants is adopted.

Work in progress - V



PS and PV terms will not contribute to gs and T terms are not included for the test, we expect that they will be relevant and will be included in the future.

Work in progress - VI

