# New energy density functionals for a better description of charge-exchange resonances

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*Phys. Rev.* **C 86 031306(R) 2012**
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*Phys. Rev.* **C 86 021302(R) 2012**

**Mini-workshop on "New achievements in Gamow-Teller nuclear transitions and decays", November 18$^{\mathrm{th}}$ 2014, Milano**

# General Motivation:

## Many-body systems and density functional theories

- Research on quantum mechanical many-body problems is essential in many areas of modern physics
  - electrons in metal, atoms in molecule, electrons in atom, nucleons in nucleus ...
- Density functional theories (DFT) (Hohenberg & Kohn 1964)
  - reduces the many-body problem of N-coupled equations of motion into N independent equations of motion via an average (mean-field) potential
  - no other method achieves comparable accuracy at the same computational cost

# General motivation:

## Nuclear DFT

- Nuclear DFT is a promising tool for investigating the ground-state and excited state properties of nuclei throughout the nuclear chart
- Since the 1970s, a lot of experience have been accumulated in implementing, adjusting, and using the DFT in nuclei.

# Motivation: Spin-isospin properties

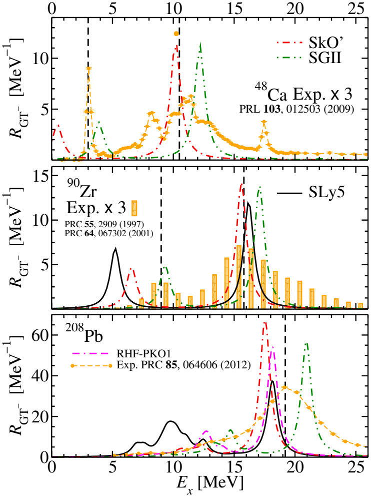
- **The H(F)+RPA method based on nuclear effective interactions of the Skyrme, Gogny or Relativistic meson-exchange types enables an effective description of the nuclear many-body problem and can be understood as an approximate realization of an EDF**
- **One of the open problems that need to be better understood and solved in current EDFs is ...**
  - the **accurate determination** of **spin-isospin properties** $\Rightarrow$ **accurate description** of e.g. charge-exchange excitations such as the **GTR**

[GT transitions ...
govern electron capture during the core-colapse of supernovæ,
matrix el. are necessary for the study of double-$\beta$ decay,
calibration of neutrino detectors ...]

# Motivation: Gamow Teller Resonance I

**Neither the strength nor the $E_x$ properly described in H(F)+RPA**

- SGII[a] $\Rightarrow$ earliest attempt to give a quantitative description of the GTR
- SkO'[b] $\Rightarrow$ accurate in ground state finite nuclear properties and improves the GTR
- Relativistic MF[c]: residual interaction modified *ad-hoc*




[a] N. Giai and H. Sagawa, Phys. Lett. B **106**, 379 (1981), [b] P.-G. Reinhard et al., Phys. Rev. C **60**, 014316 (1999), [c] H. Liang, N. Van Giai, and J. Meng, Phys. Rev. Lett. **101**, 122502 (2008), SLy5 – E. Chabanat et al., Nucl. Phys. A **635**, 231 (1998); E. Chabanat *et al.*, ibid. **643**, 441 (1998)

# Motivation: Gamow Teller Resonance II

## Quenching of the strength

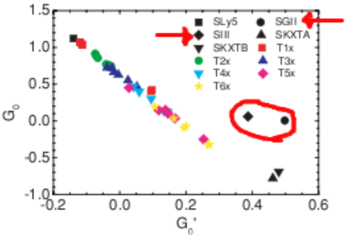
- **Experimentally**, the **GTR** exhausts **60–70%** of the **Ikeda sum rule**: $\int [R_{\mathrm{GT}^-}(E) - R_{\mathrm{GT}^+}(E)]dE = 3(N-Z)$
- To **explain** the problem, two possibilities that go beyond $(1p-1h)$ RPA correlations have been drawn:
  - the effects of the second-order configuration mixing: **2p-2h correlations**
  - within the quark model, a **n(p)** can become a **p(n)** or a $\Delta^+(\Delta^{++})$ under the action of the $\mathrm{GT}^-$ operator and since there is **no Pauli blocking for $\Delta$−h excitations** $\Rightarrow$ it may **contribute to the GTR**.
- The **experimental analysis of $^{90}$Zr** $\Rightarrow$ **quenching** ( 2/3) has to be **mainly attributed to 2p-2h** coupling and not to $\Delta$−isobar effects much smaller [T. Wakasa *et. al.*, Phys. Rev. C **55**, 2909 (1997)].
- ***$E_x$* GTR in nuclei** mainly in the region of several **tens of MeV** and the $\Delta$−h states are hundreds of MeV above the GT $\Rightarrow$ **hard to excite the $\Delta$** in the nuclear medium.

# Motivation: which gs properties are important for describing the $E_x^{\mathrm{GTR}}$?

**The study[a] of the GTR and the spin-isospin Landau-Migdal parameter $G_0'$ using several Skyrme sets,**

- concluded that $G_0'$ **is not the only important quantity** in determining the **excitation energy of the GTR**
- **spin-orbit splittings** also influences the GTR

- Empirical indications[b] suggest that $G_0' > G_0 > 0$
- **Not a very common feature** within available Skyrme forces[c]

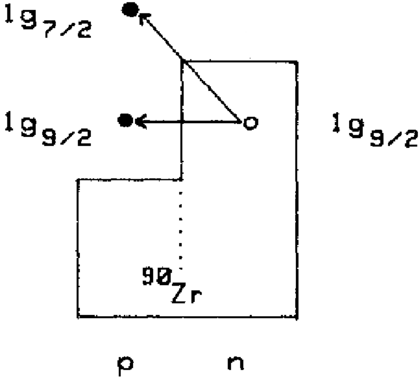



[a] M. Bender, J. Dobaczewski, J. Engel, and W. Nazarewicz, Phys. Rev. C **65**, 054322 (2002);

[b] T. Wakasa, M. Ichimura, and H. Sakai, Phys. Rev. C **72**, 067303 (2005); T. Suzuki and H. Sakai, Phys. Lett. B **455**, 25 (1999),[c] Li-Gang Cao, G. Colo, and H. Sagawa, Phys. Rev. C **81**, 044302 (2010)

# Why spin-orbit splittings are important?

$E_x^1 \approx$
$\epsilon_{\pi 1g_{7/2}} - \epsilon_{\nu 1g_{9/2}} + \epsilon^1_{\mathrm{ph}}$
$E_x^2 \approx$
$\epsilon_{\pi 1g_{9/2}} - \epsilon_{\nu 1g_{9/2}} + \epsilon^2_{\mathrm{ph}}$
$\Delta E_x \approx \boldsymbol{\Delta}\epsilon_{\pi 1g} + \Delta\epsilon_{\mathrm{ph}}$




Schematic picture of single-particle transitions involved in the Gamow Teller Resonance of $^{90}$Zr. Transitions excited by $\sigma\tau_-$ operator.

F. Osterfeld, Rev. Mod. Phys. **64**, 491 (1992)

# Table of contents:

- **New Skyrme Hartree-Fock model** that takes care of reproducing some selected proton spin-orbit splittings around the Fermi surface and the hierarchy in the Landau-Migdal parameters will be shown to improve the description of GTRs.
- **Relativistic Hartree-Fock and localized Hartree models** will be also shown to describe the GTRs with no need of *ad-hoc* readjustments in the residual interaction (as it happens within Relativistic Hartree models).

# New Skyrme interaction with improved spin-isospin properties

# Skyrme Model [ ... have a quick look!]

## Hamiltonian[a]

Includes **central tensor terms ($J^2$ terms)** due to the coupling of tensor and spin and gradients terms and **two spin-orbit parameters** (same as SkO and some SkI forces)

$$\mathcal{H} = \mathcal{K} + \mathcal{H}_0 + \mathcal{H}_3 + \mathcal{H}_{\rm eff} + \mathcal{H}_{\rm fin} + H_{\rm SO} + H_{\rm sg} + \mathcal{H}_{\rm Coul}$$

$$\begin{aligned}
\mathcal{K} &= \hbar^2\tau/2m \\
\mathcal{H}_0 &= (1/4)t_0[(2+x_0)\rho^2 - (2x_0+1)(\rho_n^2+\rho_p^2)] \\
\mathcal{H}_3 &= (1/24)t_3\rho^\alpha[(2+x_3)\rho^2 - (2x_3+1)(\rho_n^2+\rho_p^2)] \\
\mathcal{H}_{\rm eff} &= (1/8)[t_1(2+x_1)+t_2(2+x_2)]\tau\rho \\
&\quad +(1/8)[t_2(2x_2+1)-t_1(2x_1+1)](\tau_n\rho_n+\tau_p\rho_p) \\
\mathcal{H}_{\rm fin} &= (1/32)[3t_1(2+x_1)-t_2(2+x_2)](\nabla\rho)^2 \\
&\quad -(1/32)[3t_1(2x_1+1)+t_2(2x_2+1)][(\nabla\rho_n)^2+(\nabla\rho_p)^2] \\
H_{\rm SO} &= (1/2)W_0\mathbf{J}\cdot\nabla\rho + (1/2)W_0'(\mathbf{J}\cdot_{\mathbf{n}}\nabla\rho_n + \mathbf{J_p}\cdot\nabla\rho_p) \\
H_{\rm sg} &= -(1/16)(t_1x_1+t_2x_2)\mathbf{J}^2 + (1/16)(t_1-t_2)(\mathbf{J_n}^2+\mathbf{J_p}^2)
\end{aligned}$$

[a] E. Chabanat et al., Nucl. Phys. A **635**, 231 (1998); E. Chabanat *et al.*, ibid. **643**, 441 (1998)

# Fitting Protocol

**$\chi^2$ definition:** $\chi^2 = \frac{1}{N_{\rm data}} \sum_i^{N_{\rm data}} \frac{(\mathcal{O}_i^{\rm theo.} - \mathcal{O}_i^{\rm data})^2}{(\Delta\mathcal{O}_i^{\rm data})^2}$

**Landau-Migdal parameters** in infinite nuclear matter $G_0$ and $G_0'$ fixed to **0.15** and **0.35**, respectively, at $\rho_0$.

Table: Data and *pseudo*-data $\mathcal{O}_i$, adopted errors for the fit $\Delta\mathcal{O}_i$ and selected finite nuclei and EoS.

| $\mathcal{O}_i$ | $\Delta\mathcal{O}_i$ | |
|---|---|---|
| $B$ | 1.00 MeV | $^{40,48}$Ca, $^{90}$Zr, $^{132}$Sn and $^{208}$Pb |
| $r_c$ | 0.01 fm | $^{40,48}$Ca, $^{90}$Zr and $^{208}$Pb |
| $\Delta E_{\rm SO}$ | $0.04\times\mathcal{O}_i$ | $\pi 1g$ in $^{90}$Zr and $\pi 2f$ in $^{208}$Pb |
| $e_n(\rho)$ | $0.20\times\mathcal{O}_i$ | R. B. Wiringa *et al.*, PRC **38**, 1010 (1988) |

# Skyrme Aizu Milano interaction: SAMi

## Parameter set and nuclear matter properties:

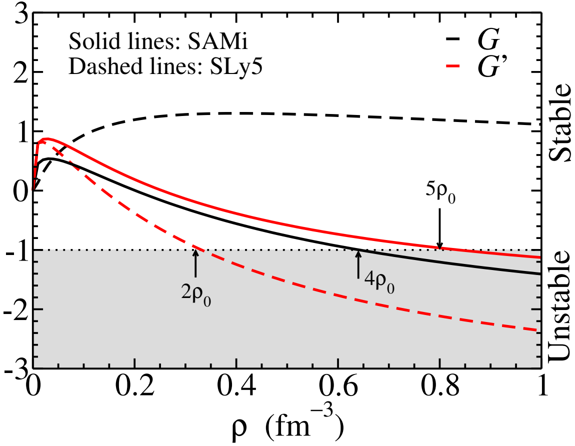
Table: SAMi parameter set and saturation properties with the estimated standard deviations inside parenthesis

| | value($\sigma$) | | | value($\sigma$) | |
|---|---|---|---|---|---|
| $t_0$ | $-1877.75(75)$ | MeV fm$^3$ | $\rho_\infty$ | 0.159(1) | fm$^{-3}$ |
| $t_1$ | 475.6(1.4) | MeV fm$^5$ | $e_\infty$ | $-15.93(9)$ | MeV |
| $t_2$ | $-85.2(1.0)$ | MeV fm$^5$ | $m^*_{\mathrm{IS}}$ | 0.6752(3) | |
| $t_3$ | 10219.6(7.6) | MeV fm$^{3+3\alpha}$ | $m^*_{\mathrm{IV}}$ | 0.664(13) | |
| $x_0$ | 0.320(16) | | $J$ | 28(1) | MeV |
| $x_1$ | $-0.532(70)$ | | $L$ | 44(7) | MeV |
| $x_2$ | $-0.014(15)$ | | $K_\infty$ | 245(1) | MeV |
| $x_3$ | 0.688(30) | | $G_0$ | 0.15 | (fixed) |
| $W_0$ | 137(11) | | $G'_0$ | 0.35 | (fixed) |
| $W'_0$ | 42(22) | | | | |
| $\alpha$ | 0.25614(37) | | | | |

# SAMi: spin and spin-isospin instabilities in NM

Imposing that spin and isospin dof at the Fermi surface are stable under generalized deformations [S.-O. Bäckman *et al.*, Nucl. Phys. A **321**, 10 (1979)]

$$1 + G_0 > 0 \qquad 1 + G_0' > 0$$



# Results

## Equation of State: SAMi vs *ab*–initio calculations

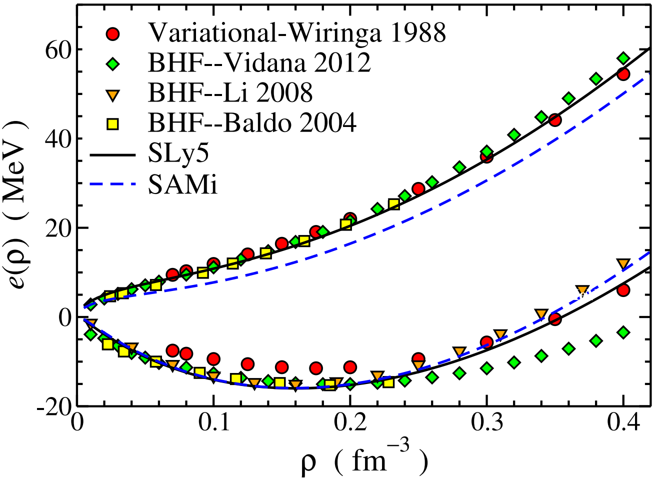



Figure: Neutron and symmetric matter EoS as predicted by the HF SAMi (dashed line) and SLy5 (solid line) interactions and by the benchmark microscopic calculations of R. B. Wiringa *et al.*, PRC **38**, 1010 (1988) (circles). State-of-the-art BHF calculations are shown by diamonds I. Vidaña, private communication, triangles Z. H. Li *et al.*, Phys. Rev. C **77**, 034316 (2008) and squares M. Baldo *et al.*, Nucl. Phys. A **736**, 241 (2004).

# Results

## Finite Nulcei: spherical double-magic nuclei

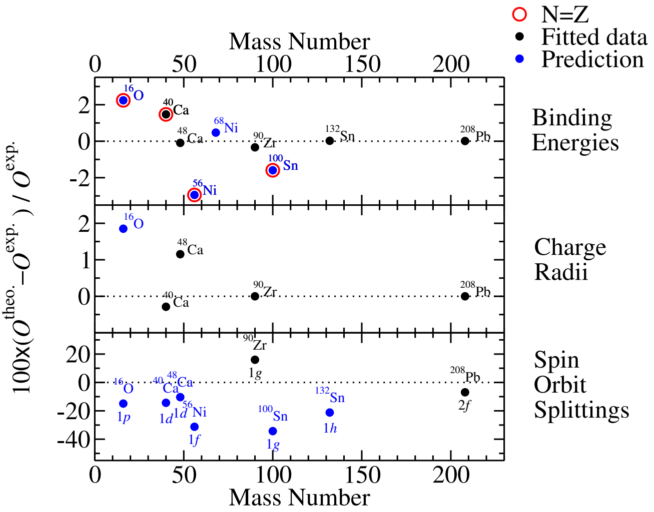



Figure: Finite nuclei properties as predicted by the HF SAMi (black circles) and some predictions (blue circles) for spherical double-magic nuclei. Experimental data taken from Refs. G. Audi *et al.*, NPA **729**, 337 (2003), I. Angeli, ADNDT **87**, 185 (2004), M. Zalewski *et al.*, PRC **77**, 024316 (2008)

# Results

## Giant Monopole and Dipole Resonances in $^{208}$Pb

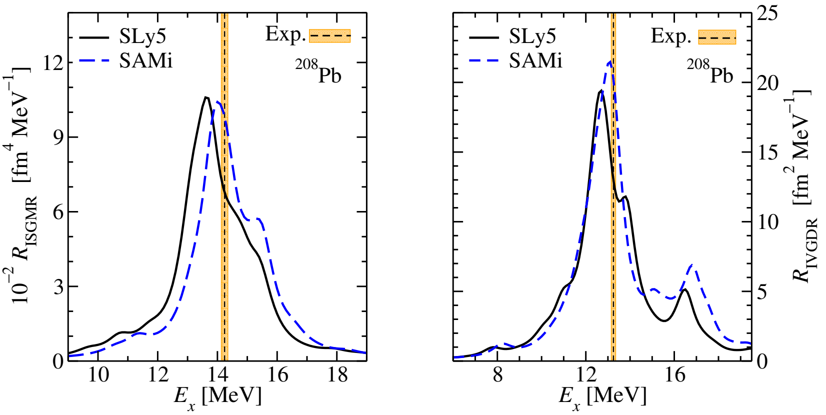
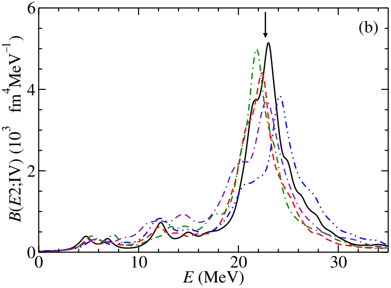
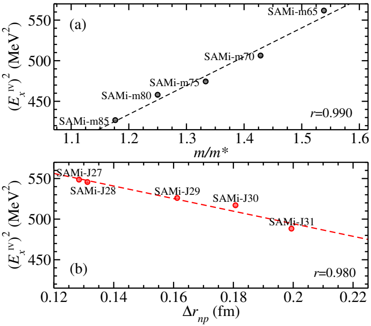



Figure: Strength function at the relevant excitation energies in $^{208}$Pb as predicted by SLy5 and the SAMi interaction for GMR and GDR. A Lorentzian smearing parameter equal to 1 MeV is used. Experimental data for the centroid energies are also shown: $E_c(GMR) = 14.24 \pm 0.11$ MeV [D. H. Youngblood, et al., Phys. Rev. Lett. **82**, 691 (1999)] and $E_c(GDR) = 13.25 \pm 0.10$ MeV [N. Ryezayeva et al., Phys. Rev. Lett. **89**, 272502 (2002)].

# Results

## IV Giant Quadrupole Resonance in $^{208}$Pb




Recently measured by using polarized $\gamma-$ray beams **(as in future facility in Romania)** [S. S. Henshaw, *et al.* Phys. Rev. Lett. 107, 222501]

Simple model based on the HO (B&M):

$$E_x^{\mathrm{IV}} \approx 2\left[\frac{\left(E_x^{\mathrm{IS}}\right)^2}{2} + 2\frac{\varepsilon_{\mathrm{F}\infty}^2}{A^{2/3}}\left(\frac{3S(\rho_A)}{\varepsilon_{\mathrm{F}\infty}} - 1\right)\right]^{1/2}$$

Phys. Rev. C 87, 034301 (2013).

# Results

## Gamow Teller Resonance in $^{48}$Ca, $^{90}$Zr and $^{208}$Pb

Operator:
$\sum_{i=1}^{A} \sigma(i)\tau_{\pm}(i)$

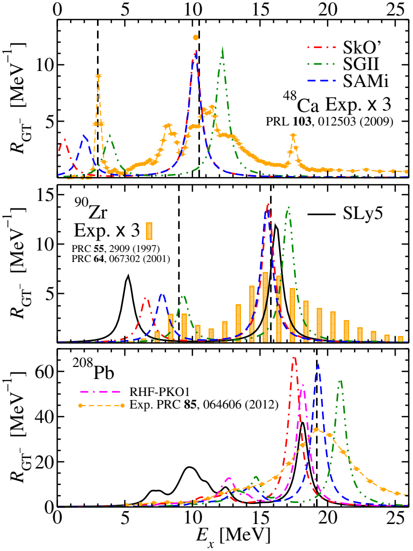



**Figure:** Gamow Teller strength distributions in $^{48}$Ca (upper panel), $^{90}$Zr (middle panel) and $^{208}$Pb (lower panel) as measured in the experiment [T. Wakasa *et al.*, Phys. Rev. C **55**, 2909 (1997), K. Yako *et al.*, Phys. Rev. Lett. **103**, 012503 (2009), A. Krasznaborkay *et al.*, Phys. Rev. C **64**, 067302 (2001), H. Akimune *et al.*, Phys. Rev. C **52**, 604 (1995) and T. Wakasa et al., Phys. Rev. C **85**, 064606 (2012)] and predicted by SLy5, SkO', SGII and SAMi forces.

# Results

## Spin Dipole Resonances in $^{90}$Zr and $^{208}$Pb

Operator:
$\sum_{i=1}^{A} \sum_{M} \tau_{\pm}(i) r_i^L \left[ Y_L(\hat{r}_i) \otimes \sigma(i) \right]_{JM}$
Sum Rule:
$\int [R_{\mathrm{SD}^-}(E) - R_{\mathrm{SD}^+}(E)] \mathrm{d}E =$
$\frac{9}{4\pi} (N \langle r_n^2 \rangle - Z \langle r_p^2 \rangle)$

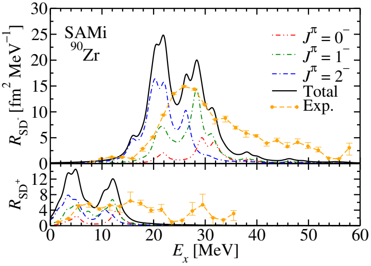



**Figure:** Spin Dipole strength distributions in $^{90}$Zr as a function of the excitation energy $E_x$ in the $\tau_-$ channel (upper panel) and $\tau_+$ channel (lower panel) measured in the experiment [K. Yako *et al.*, Phys. Rev. C **74**, 051303(R) (2006)] and predicted by SAMi. Multipole decomposition is also shown. A Lorentzian smearing parameter equal to 2 MeV is used.

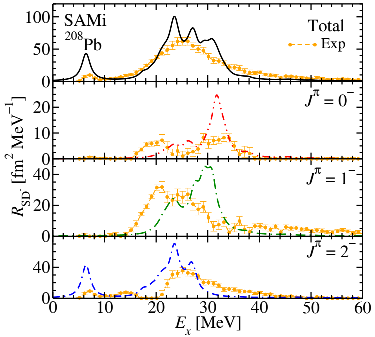



**Figure:** SDR strength distributions for $^{208}$Pb in the $\tau_-$ channel from experiment [T. Wakasa *et al.*, Phys. Rev. C **85**, 064606 (2012)] and SAMi calculations. Total and multipole decomposition of the SDR strength are shown: total (upper panel), $J^\pi = 0^-$ (middle-upper panel), $J^\pi = 1^-$ (middle-lower panel) and $J^\pi = 2^-$ (lower panel). A Lorentzian smearing parameter equal to 2 MeV is used.

# Conclusions:

- we have **successfully determined a new Skyrme** energy density functional which **accounts** for the most relevant quantities in order to improve the description of **charge-exchange nuclear resonances**:
  - the **hierarchy** and **positive values** of the spin and spin-isospin Landau-Migdal parameters $G_0$ **and** $G_0'$
  - the **proton spin-orbit splittings** of different **high angular momenta** single-particle levels
- the **GTR** in $^{48}$Ca and the **GTR**, **IAR**, and **SDR** in $^{90}$**Zr** and $^{208}$**Pb** are predicted with **good accuracy by SAMi**
- **SAMi** does **not deteriorate** the description of other **nuclear observables**
- **applicability in nuclear physics and astrophysics**

# Relativistic HF and localized H models

# Covariant density functional theory – heavy meson exchange

- Effective Lagrangian density

$$\mathcal{L} = \bar{\psi}\left[i\gamma^\mu\partial_\mu - M - g_\sigma\sigma - \gamma^\mu\left(g_\omega\omega_\mu + g_\rho\vec{\tau}\cdot\vec{\rho}_\mu + e\frac{1-\tau_3}{2}A_\mu\right)\right]\psi$$
$$+\frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu$$
$$-\frac{1}{4}\vec{R}_{\mu\nu}\cdot\vec{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}^\mu\cdot\vec{\rho}_\mu - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

- System Hamiltonian

$$\mathcal{H} = \mathcal{T}^{00} = \frac{\partial\mathcal{L}}{\partial\dot{\phi}_i}\dot{\phi}_i - \mathcal{L}$$

- Energy functional of the system

$$E = \langle\Phi_0|\,H\,|\Phi_0\rangle = E_k + E_\sigma^D + E_\omega^D + E_\rho^D + E_A^D + E_\sigma^E + E_\omega^E + E_\rho^E + E_A^E$$

# From meson-exchange to zero-range relativistic forces

- Yukawa propagators of the mesons

$$D_i(\boldsymbol{r},\boldsymbol{r}') = \frac{1}{4\pi}\frac{e^{-m_i|\boldsymbol{r}-\boldsymbol{r}'|}}{|\boldsymbol{r}-\boldsymbol{r}'|}, \qquad D_i(\boldsymbol{q}) = \frac{1}{m_i^2+\boldsymbol{q}^2} \tag{1}$$

for $m_i \gg q$,

$$D_i(\boldsymbol{q}) \approx \frac{1}{m_i^2} - \frac{\boldsymbol{q}^2}{m_i^4} + \cdots \quad \Rightarrow \quad D_i(\boldsymbol{r},\boldsymbol{r}') \approx \frac{1}{m_i^2}\delta(\boldsymbol{r}-\boldsymbol{r}') \tag{2}$$

within the zero-order approximation.

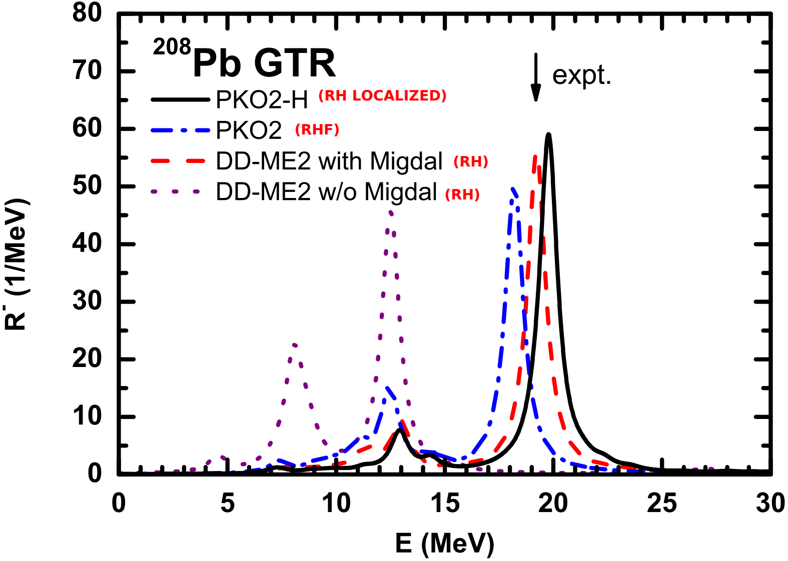
- Zero-range reduction of meson-nucleon couplings

$$\alpha_S^{\mathrm{HF}} = -\frac{g_\sigma^2}{m_\sigma^2}, \qquad \alpha_V^{\mathrm{HF}} = \frac{g_\omega^2}{m_\omega^2}, \qquad \alpha_{tV}^{\mathrm{HF}} = \frac{g_\rho^2}{m_\rho^2}, \tag{3}$$

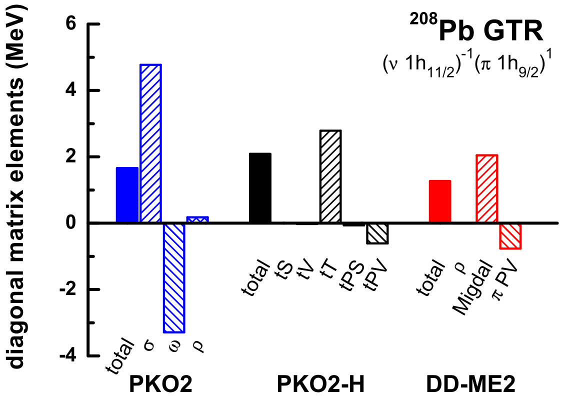
# Fierz transformation: from HF to H

$$
\begin{aligned}
\alpha_S^{\mathrm{H}} &= +\frac{7}{8}\alpha_S^{\mathrm{HF}} - \frac{4}{8}\alpha_V^{\mathrm{HF}} - \frac{12}{8}\alpha_{tV}^{\mathrm{HF}} \\
\alpha_{tS}^{\mathrm{H}} &= -\frac{1}{8}\alpha_S^{\mathrm{HF}} - \frac{4}{8}\alpha_V^{\mathrm{HF}} + \frac{4}{8}\alpha_{tV}^{\mathrm{HF}} \\
\alpha_V^{\mathrm{H}} &= -\frac{1}{8}\alpha_S^{\mathrm{HF}} + \frac{10}{8}\alpha_V^{\mathrm{HF}} + \frac{6}{8}\alpha_{tV}^{\mathrm{HF}} \\
\alpha_{tV}^{\mathrm{H}} &= -\frac{1}{8}\alpha_S^{\mathrm{HF}} + \frac{2}{8}\alpha_V^{\mathrm{HF}} + \frac{6}{8}\alpha_{tV}^{\mathrm{HF}} \\
\alpha_T^{\mathrm{H}} &= -\frac{1}{16}\alpha_S^{\mathrm{HF}} \\
\alpha_{tT}^{\mathrm{H}} &= -\frac{1}{16}\alpha_S^{\mathrm{HF}} \\
\alpha_{PS}^{\mathrm{H}} &= -\frac{1}{8}\alpha_S^{\mathrm{HF}} + \frac{4}{8}\alpha_V^{\mathrm{HF}} + \frac{12}{8}\alpha_{tV}^{\mathrm{HF}} \\
\alpha_{tPS}^{\mathrm{H}} &= -\frac{1}{8}\alpha_S^{\mathrm{HF}} + \frac{4}{8}\alpha_V^{\mathrm{HF}} - \frac{4}{8}\alpha_{tV}^{\mathrm{HF}} \\
\alpha_{PV}^{\mathrm{H}} &= +\frac{1}{8}\alpha_S^{\mathrm{HF}} + \frac{2}{8}\alpha_V^{\mathrm{HF}} + \frac{6}{8}\alpha_{tV}^{\mathrm{HF}} \\
\alpha_{tPV}^{\mathrm{H}} &= +\frac{1}{8}\alpha_S^{\mathrm{HF}} + \frac{2}{8}\alpha_V^{\mathrm{HF}} - \frac{2}{8}\alpha_{tV}^{\mathrm{HF}}
\end{aligned}
$$

# Results on the GTR in $^{208}$Pb

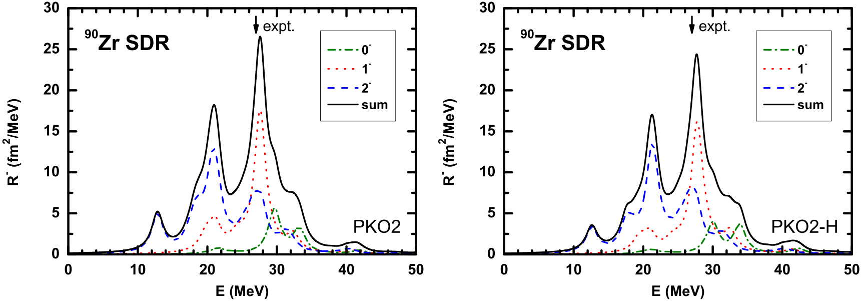


# Physical mechanisms of GTR




- RHF+RPA (PKO2): exch. terms in $\sigma$ and $\omega \Rightarrow$ GTR
- Localized RH+RPA (PKO2-H): $tT$ and $tPV \Rightarrow$ GTR
- RH+RPA (DD-ME2): free $\pi$ residual interaction + Migdal term fitted to data $\Rightarrow$ GTR

# Results on SDR in $^{90}$Zr




expt: Yako, Sagawa, Sakai, *PRC* **74**, 051303 (2006)

## Summary

- Fully self-consistent RHF+RPA (with no *ad hoc* fitted terms) is able to reproduce fairly well experimental data on IAR, GTR and SDR.
- New method is proposed to take into account the Fock terms in local covariant density functionals that mantains the essential features of RHF+RPA.
- This opens a new door for the development of nuclear local (Hartree) zero-range covariant density functionals with improved isovector properties in the future.

# Thank you!

**Work in collaboration with: G. Colò, H. Liang, J. Meng, P. Ring, H. Sagawa and P. Zhao**