The nuclear symmetry energy and the breaking of the isospin symmetry: how do they reconcile with each other?

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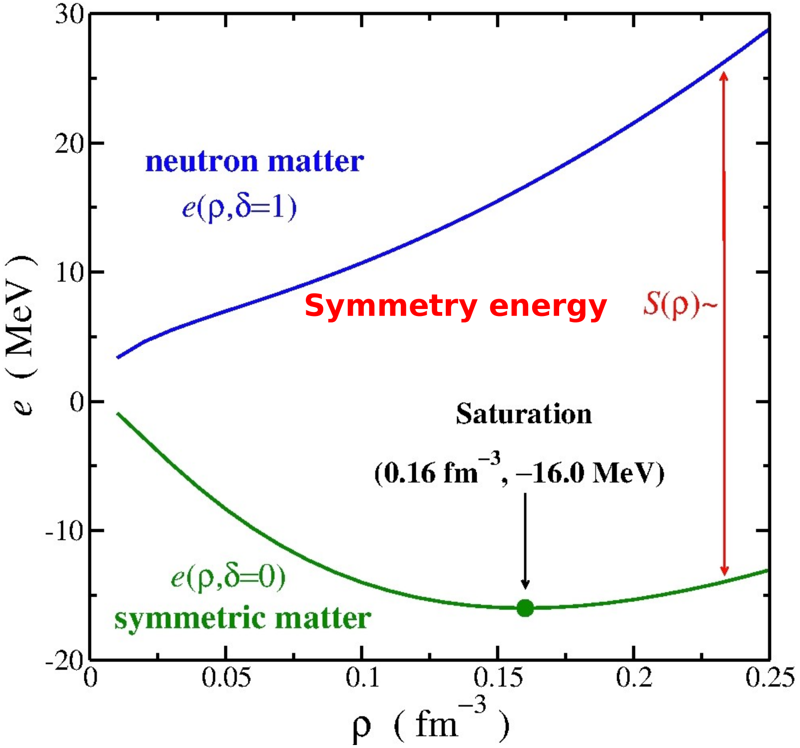
Nuclear EoS

Unpolarized **nuclear matter** at zero temperature (10^{10}K→1MeV) can be defined as the **energy** (e) per **nucleon** (A) as a function of the **neutron** and **proton densities** as (*isospin conserving*):

$$\boxed{e(\rho,\delta) = e(\rho,0) + S(\rho)\delta^2} + \mathcal{O}[\delta^4] \quad \text{where } \rho = \rho_n + \rho_p \text{ and } \delta = \frac{\rho_n - \rho_p}{\rho}$$



It is customary to **expand e(ρ,δ)** around nuclear **saturation density** $\rho_0 \sim 0.16$ fm^{-3}

$$e(\rho,0) = e(\rho_0,0) + \frac{1}{2}K_0 x^2 + \mathcal{O}[\rho^3] \text{ where } x = \frac{\rho-\rho_0}{3\rho_0}$$

$$S(\rho) = J + Lx + \frac{1}{2}K_{\text{sym}} x^2 + \mathcal{O}[\rho^3,\delta^4]$$

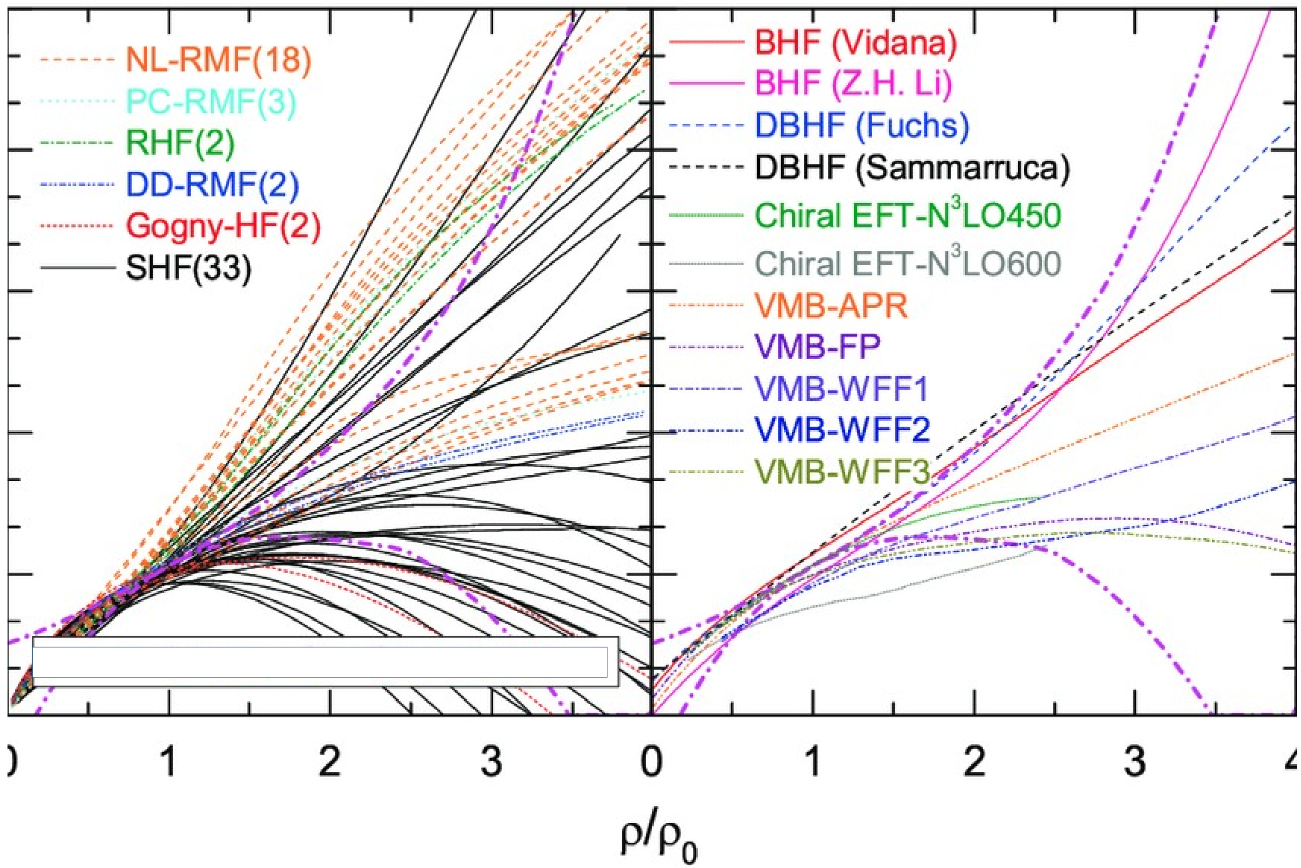
K_0 → how **compressible** is symmetric matter at ρ_0

J → **penalty energy** for converting all **protons into neutrons** from symmetric matter at ρ_0

L → **neutron pressure** in neutron matter at ρ_0

Symmetry energy

Symmetry energy not well constrained



Towards understanding astrophysical effects of nuclear symmetry energy
Bao-An Li, Plamen G. Krastev, De-Hua Wen & Nai-Bo Zhang EPJ A 55, 117 (2019)

From Heaven: Neutron Star Mass

Nuclear models that account for different nuclear properties on **Earth** predict a large **variety** of **Neutron Star Mass-Radius** relations → **<u>Observation of a 2M_{sun} has constrained nuclear models.</u>**

Tolman–Oppenheimer–Volkoff equation (sph. sym.):

$$\frac{dM(r)}{dr} = 4\pi r^2 \mathcal{E}(r);$$

$$\frac{dP}{dr} = -G\frac{\mathcal{E}(r)M(r)}{r^2}\left[1+\frac{P(r)}{\mathcal{E}(r)}\right]\left[1+\frac{4\pi r^3 P(r)}{M(r)}\right]\left[1-\frac{2GM(r)}{r}\right]^{-1}$$

$\mathcal{E}(r) \rightarrow$ degeneracy pressure from neutrons $\rightarrow M_{\max} = 0.7 M_{\text{sun}}$

Nuclear Physics input is fundamental

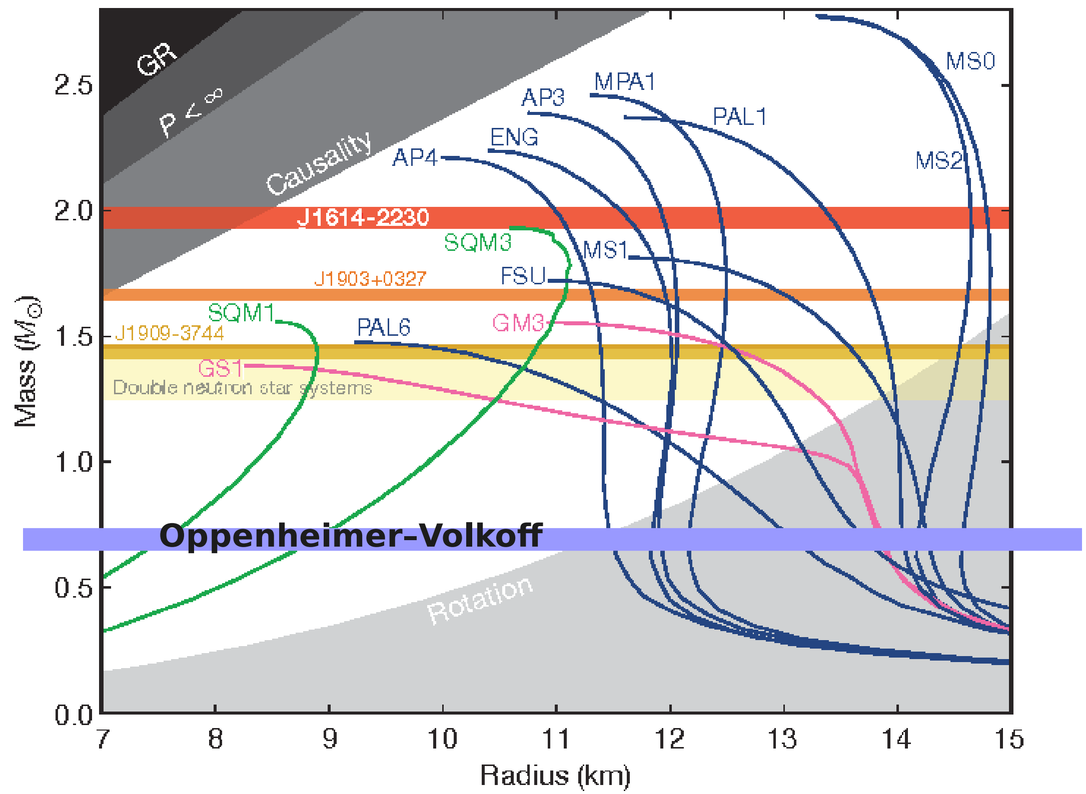
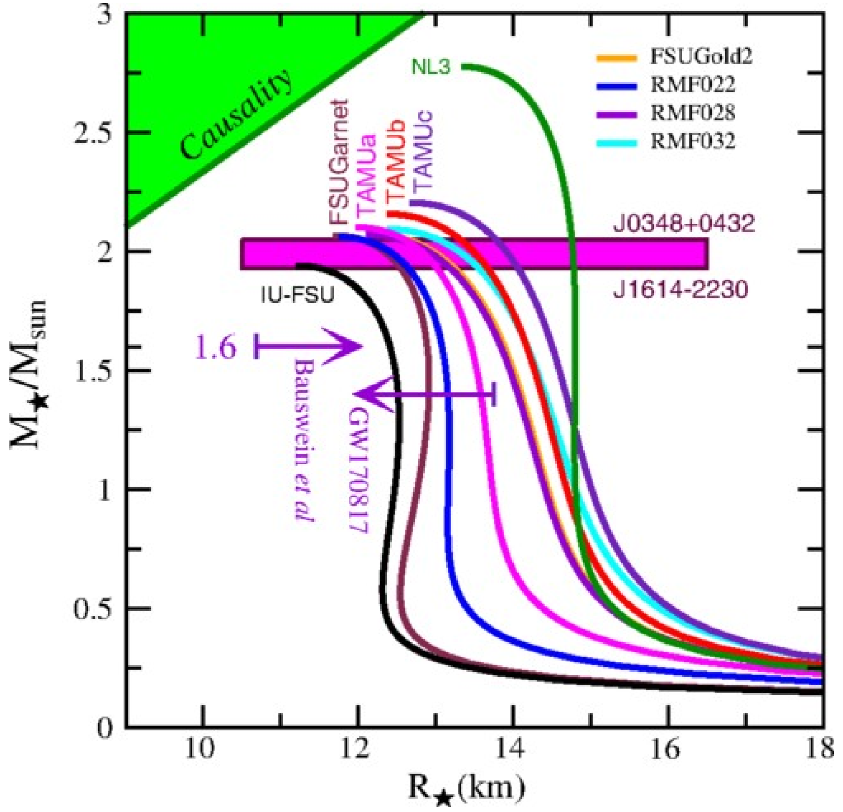


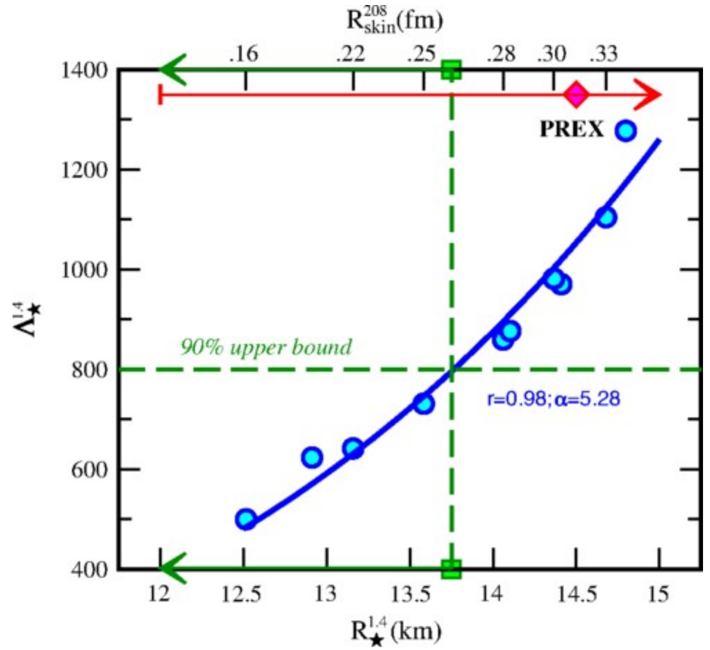
Figure 3 | Neutron star mass–radius diagram. The plot shows non-rotating

A two-solar-mass neutron star measured using Shapiro delay - P. B. Demorest, T. Pennucci, S. M. Ransom, M. S. E. Roberts & J. W. T. Hessels - Nature volume 467, 1081–1083(2010)

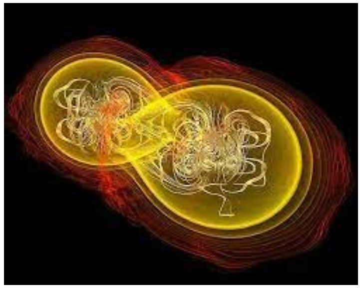
From Heaven: Gravitational wave signal from a binary neutron star merger

GW170817 from the binary neutron star merger → **constraint** neutron star **radius** and, thus, the **nuclear EoS**





Tidal deformability (Λ) is a quadrupole deformation inferred from **GW signal →** proportional to **restoring force.** Hence, sensitive to the **nuclear EoS**



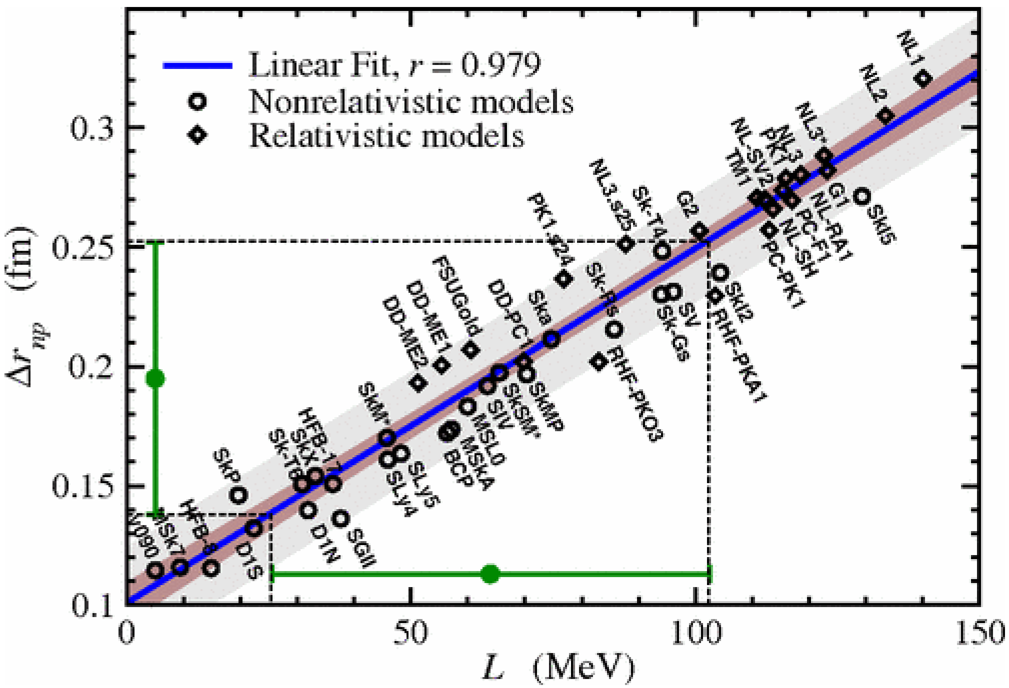
Neutron Skins and Neutron Stars in the Multimessenger Era
F. J. Fattoyev, J. Piekarewicz, and C. J. Horowitz Phys. Rev. Lett. 120, 172702 (2018)

From Heaven & Earth: neutron skin and the Radius of a Neutron Star

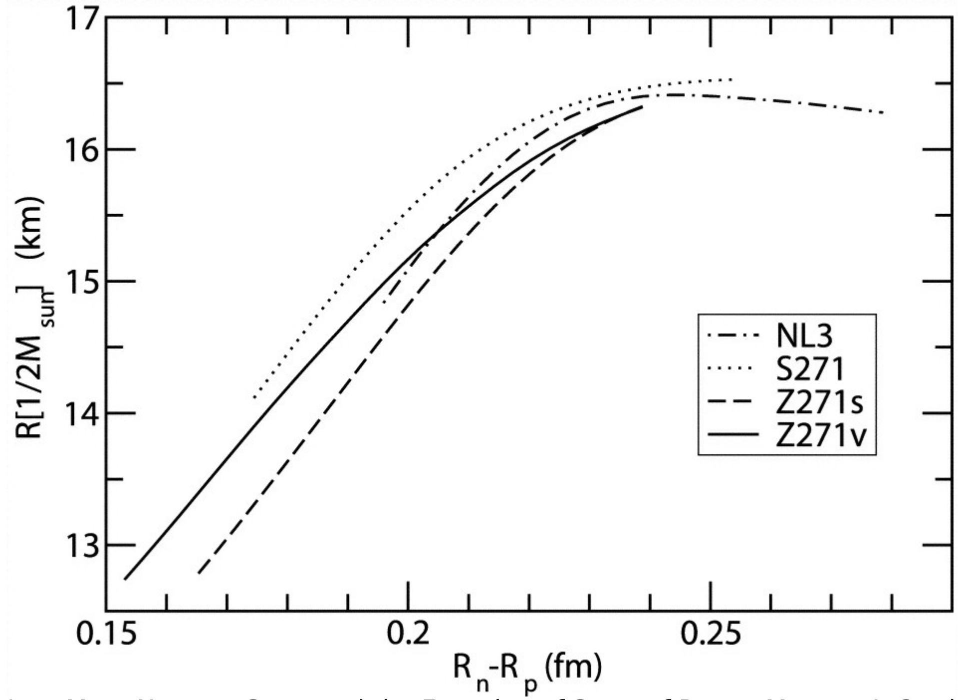
Both, the **neutron skin thickness** ($\Delta r_{np}=r_n-r_p$) in neutron rich nuclei and the **radius** of a **neutron star** are related to the **neutron pressure** in infinite matter. The former around ρ_0 (L) while the latter at larger densities.

$$\Delta r_{np} \approx \frac{1}{12}\frac{N-Z}{A}\frac{R}{J}L$$

→ For **small neutron stars**, that is, for small central densities: nuclear models predict a **linear** relation between **R** and **Δr_{np}**



Neutron Skin of 208Pb, Nuclear Symmetry Energy, and the Parity Radius Experiment X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda Phys. Rev. Lett. 106, 252501 (2011)



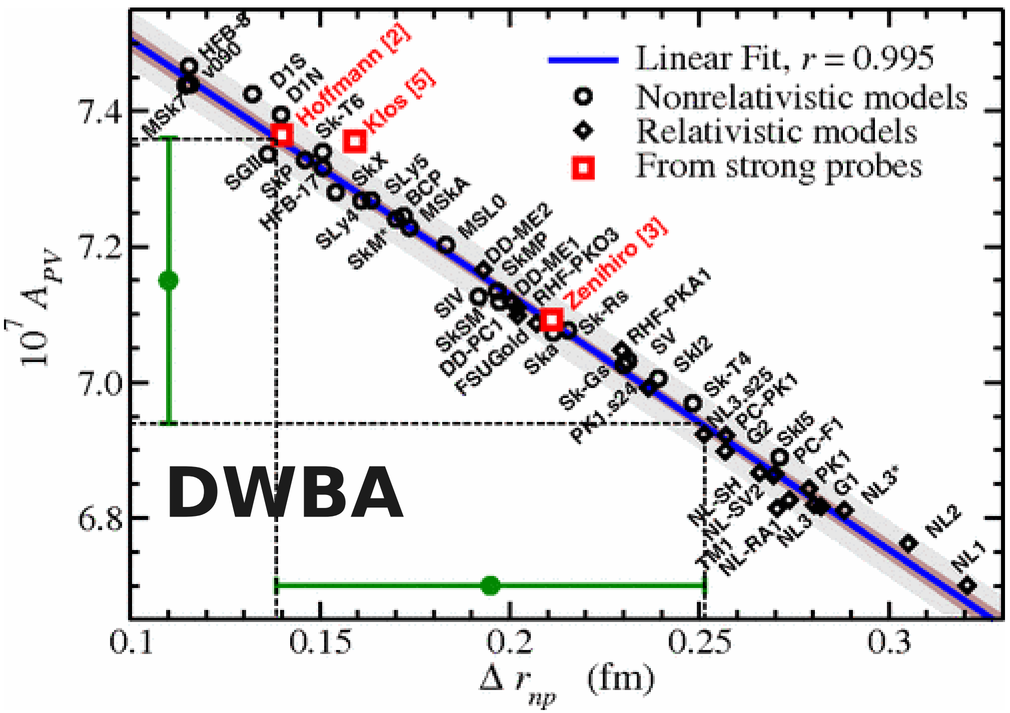
Low-Mass Neutron Stars and the Equation of State of Dense Matter - J. Carriere, C. J. Horowitz, and J. Piekarewicz - The Astrophysical Journal, 593 (2003) 463

From Earth: Parity violating electron scattering and the neutron skin

Polarized electron-Nucleus scattering:

→ In good approximation, the weak interaction probes the neutron distribution in nuclei while Coulomb interaction probes the proton distribution

→ Different experimental efforts @ Jlab (USA) & MAMI (Germany)



Neutron Skin of 208Pb, Nuclear Symmetry Energy, and the Parity Radius Experiment
X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda Phys. Rev. Lett. 106, 252501 (2011)

→ **Electrons** interact by **exchanging** a **γ** (couples to **p**) or a **Z_0** boson (couples to **n**)

→ **Ultra-relativistic electrons**,depending on their helicity (±), will interact with the nucleus seeing a slightly different potential: **Coulomb** ± **Weak**

$$A_{pv} = \frac{d\sigma_+/d\Omega - d\sigma_-/d\Omega}{d\sigma_+/d\Omega + d\sigma_-/d\Omega} \sim \frac{\text{Weak}}{\text{Coulomb}}$$

→ Main **unknown** is **ρ_n**

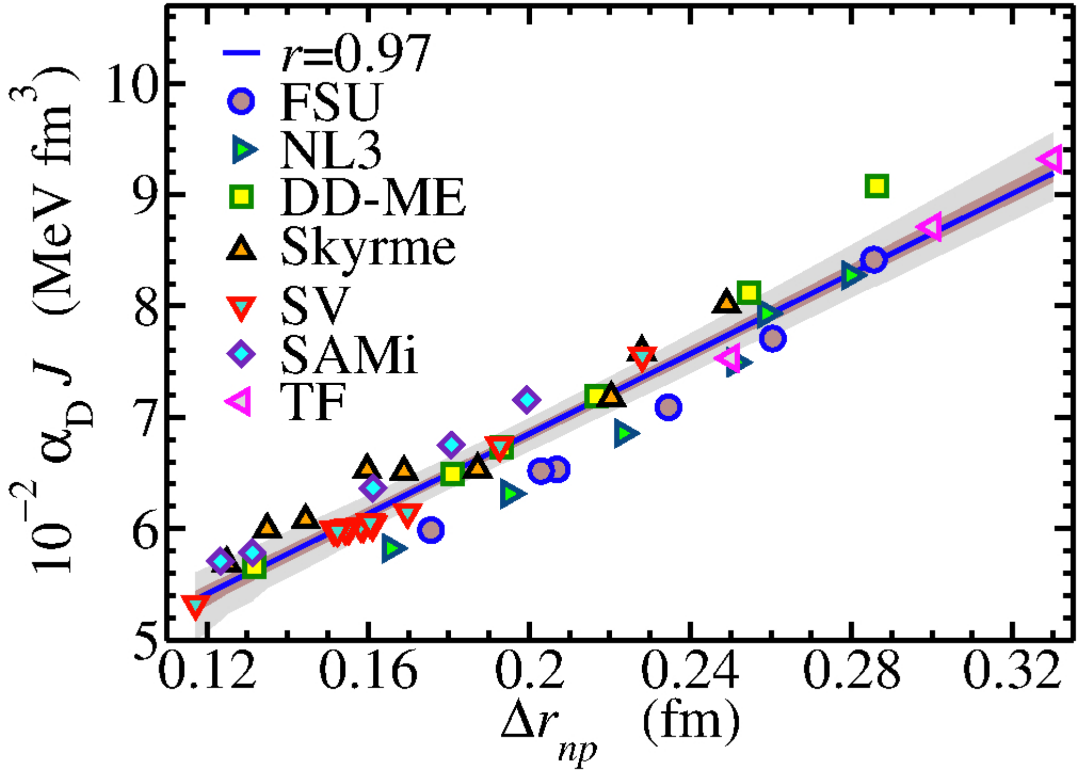
→ In **PWBA** for small momentum transfer **q**:

$$A_{pv} = \frac{G_F q^2}{4\sqrt{2}\pi\alpha}\left(1 - \frac{q^2 r_p^2}{3F_p(q)}\right)\Delta r_{np}$$

From Earth: dipole polarizability and neutron skin

The dipole **polarizability** measures the **tendency** of the nuclear **charge** distribution to be **distorted.**

From a macroscopic point of view **α ~ (electric dipole moment)/(external electric field)**



→ Using the **dielectric theorem**: the polarizability can be computed from the expectation value of the Hamiltonian in the constrained ground state H′=H+λD

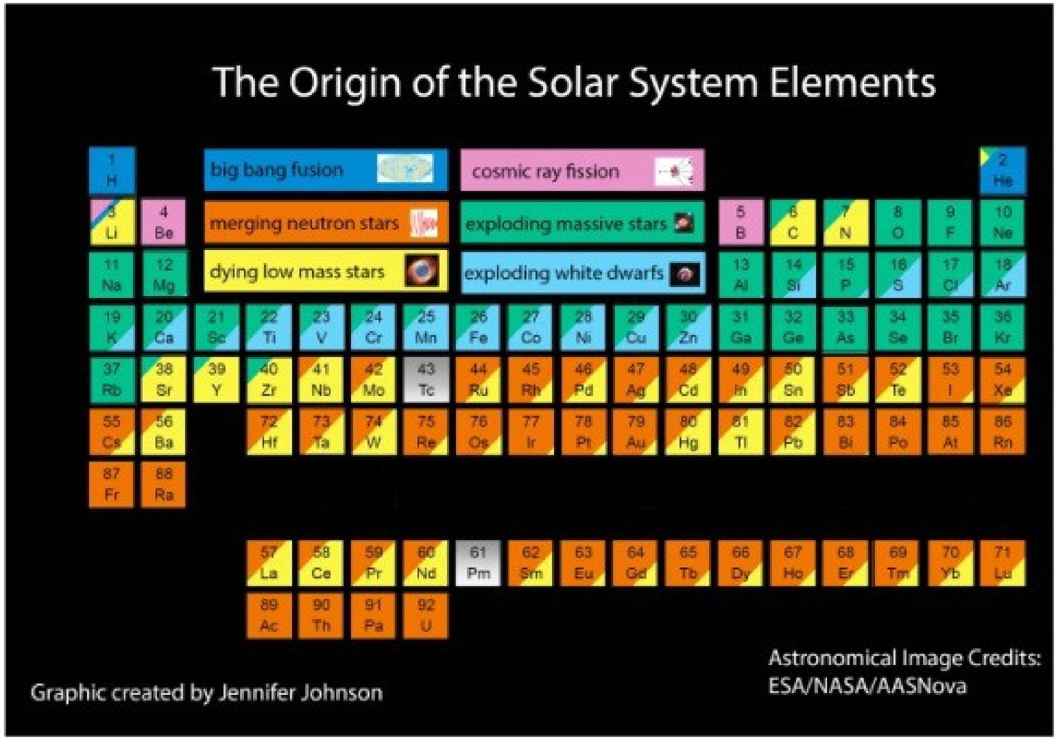
→ For guidance assuming the **Droplet model** for H, one would find:

$$\alpha_D \approx \frac{\pi e^2}{54}\frac{\langle r^2\rangle}{J}A\left(1+\frac{5}{2}\frac{\Delta r_{np}-\Delta r_{np}^{\mathrm{surf}}-\Delta r_{np}^{\mathrm{Coul}}}{\langle r^2\rangle^{1/2}(I-I_{\mathrm{Coul}})}\right)$$

Electric dipole polarizability in 208Pb: Insights from the droplet model - X. Roca-Maza, M. Brenna, G. Colò, M. Centelles, X. Viñas, B. K. Agrawal, N. Paar, D. Vretenar, and J. Piekarewicz

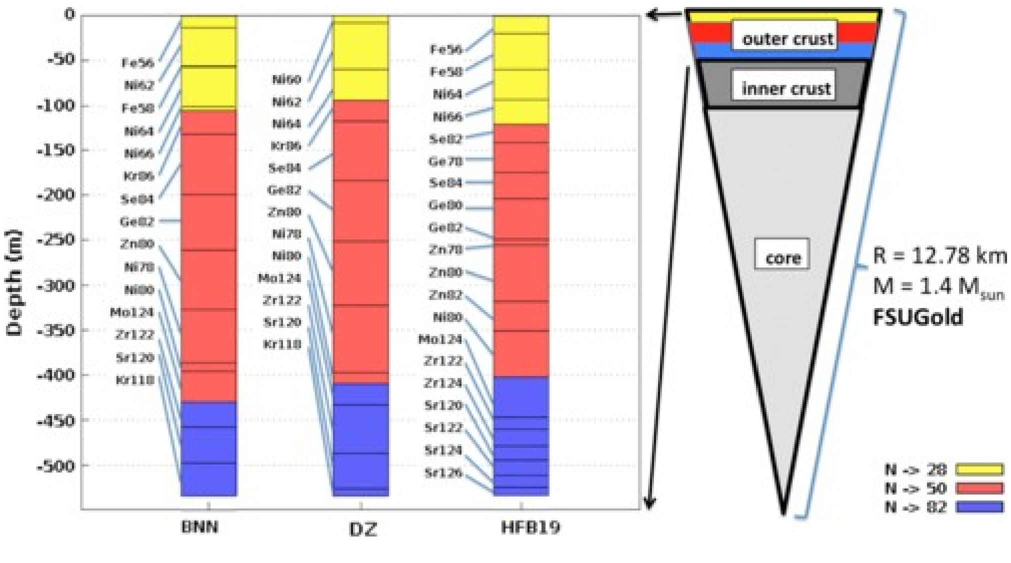
Phys. Rev. C 88, 024316 (2013)

From Heaven: Origin of elements



The optical counterpart from the binary neutron star merger produced about **10^{29}kg** of **heavy elements** → binary **NS merger** play a role in the **r-process**

The **crust** of a **NS** is made of very **exotic neutron rich nuclei,** stable only due to the extreme conditions (large densities). **Different nuclear models predict different compositions**



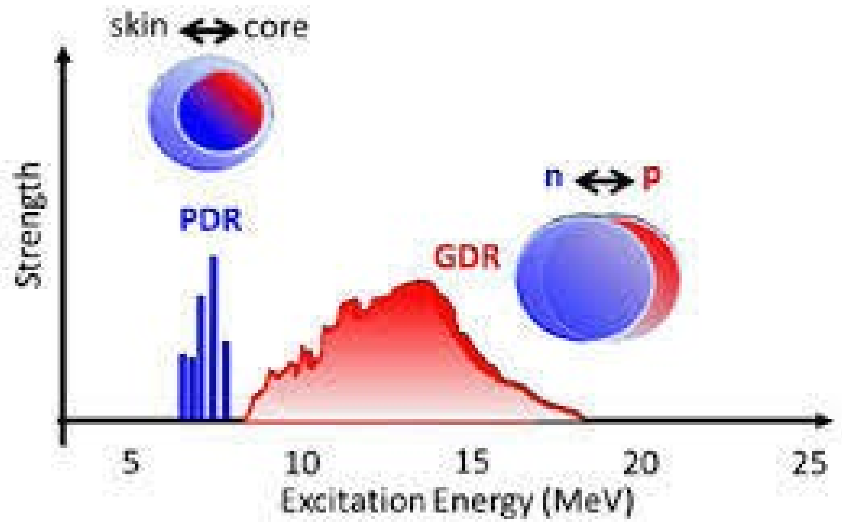
Nuclear mass predictions for the crustal composition of neutron stars: A Bayesian neural network approach R. Utama, J. Piekarewicz, and H. B. Prosper, Phys. Rev. C 93, 014311 (2016)

From Heaven & Earth: low energy dipole response and nucleosynthesis

The **largest** the **neutron pressure** among neutrons (~L), the more the **excess neutrons** (~skin) are ***"pushed out"*** in the **outermost** part of the **nucleus →** spatial ***decorrelation*** of some of those neutrons with the nucleons in the core produces **larger low lying** responses**.**

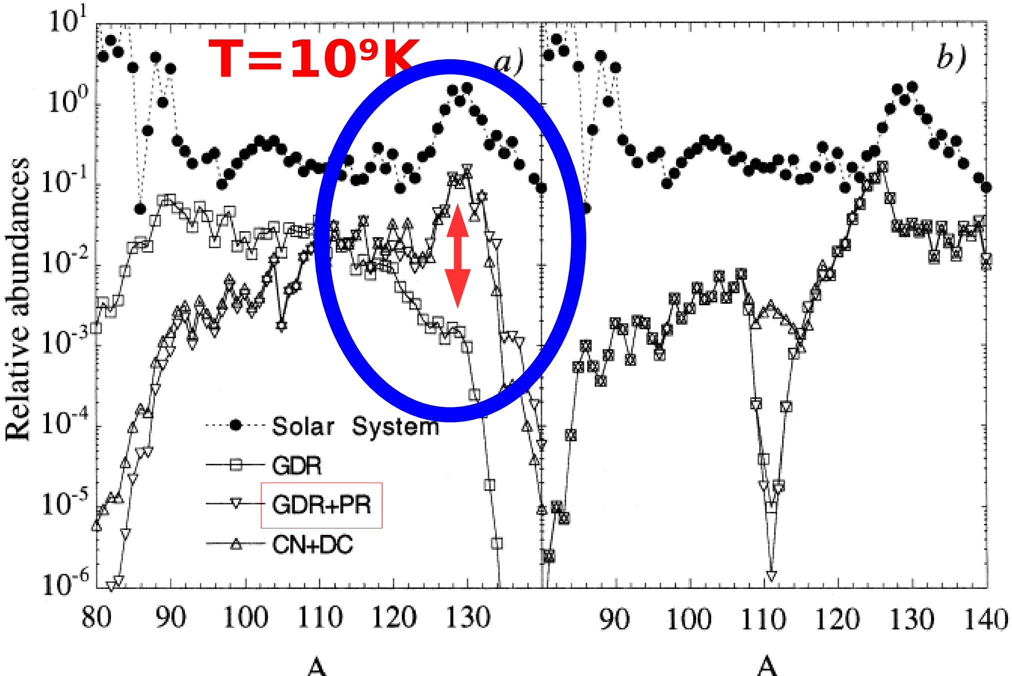
GDR=Giant Dipole Resonance

PDR= Pygmy Dipole Resonance



(nature of PDR still under debate)

Radiative neutron captures by neutron-rich nuclei and the r-process nucleosynthesis
S. Goriely, Phys.Lett.B 436 (1998) 10-18

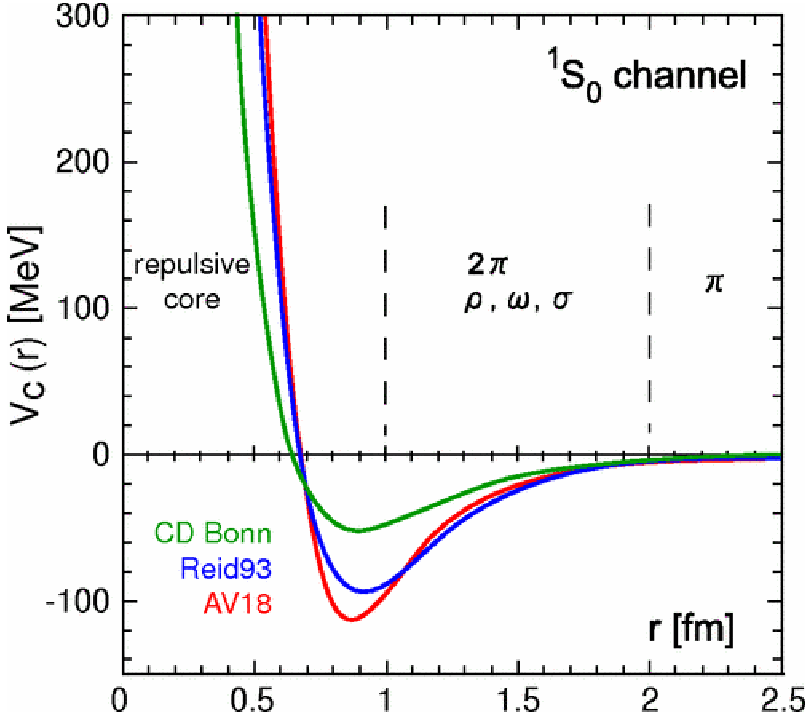


Low energy dipole strength in neutron-rich nuclei influences the neutron capture cross section and, thus, the r-process nucleosynthesis

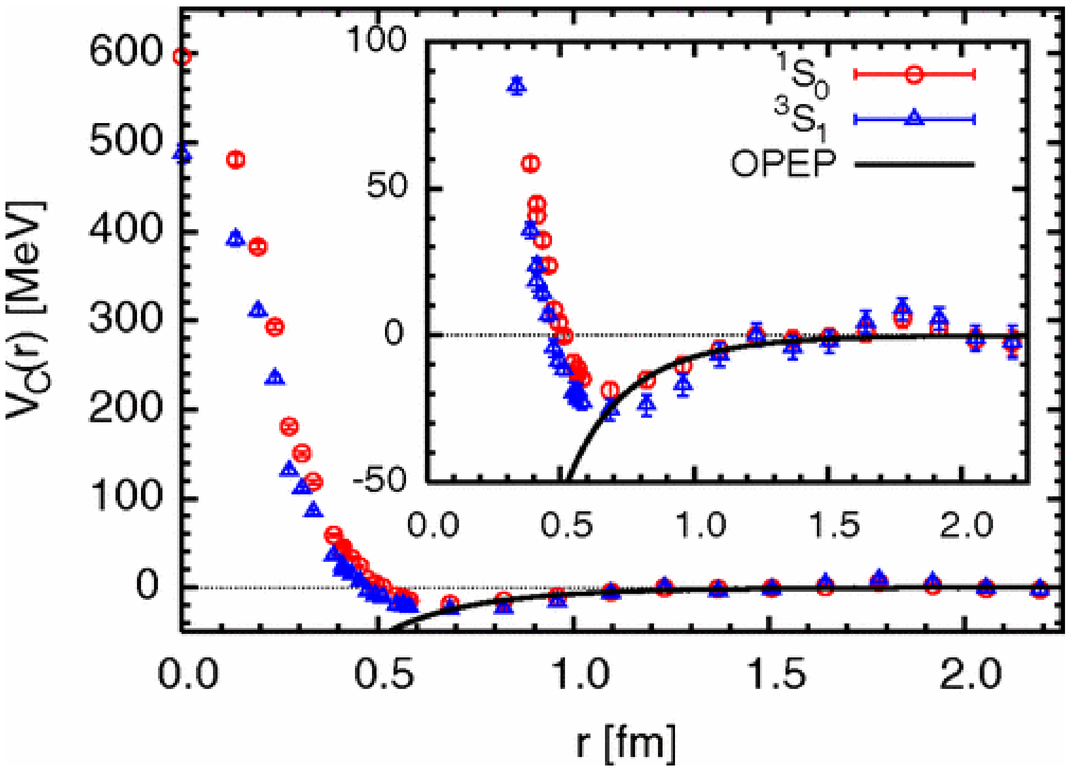
Nuclear Many-Body Problem

Underlying interaction: the “so called” **residual strong interaction** = **nuclear force** has **not** been **derived yet** (with the precision needed) from first principles as **QCD** is **non-perturbative** at the **low-energies** relevant for the description of nuclei.

Phenomenological



Lattice QCD (m_π/m_ρ~0.6)



Nuclear Force from Lattice QCD - N. Ishii, S. Aoki, and T. Hatsuda
Phys. Rev. Lett. 99, 022001 (2007)

Hohenberg-Kohn theorems

P.Hohenberg, W. Kohn, Phys. Rev. 136, B864 (1964)

→ Assuming a system of **interacting fermions** in a confining **external potential**, there exist a **universal** functional **F[ρ]** of the fermion density **ρ**:

$$E[\rho] = \langle \Psi | T + V + V_{\text{ext}} | \Psi \rangle = F[\rho] + \int V_{\text{ext}}(r)\rho(r) d\vec{r}$$

→ and it can be shown that

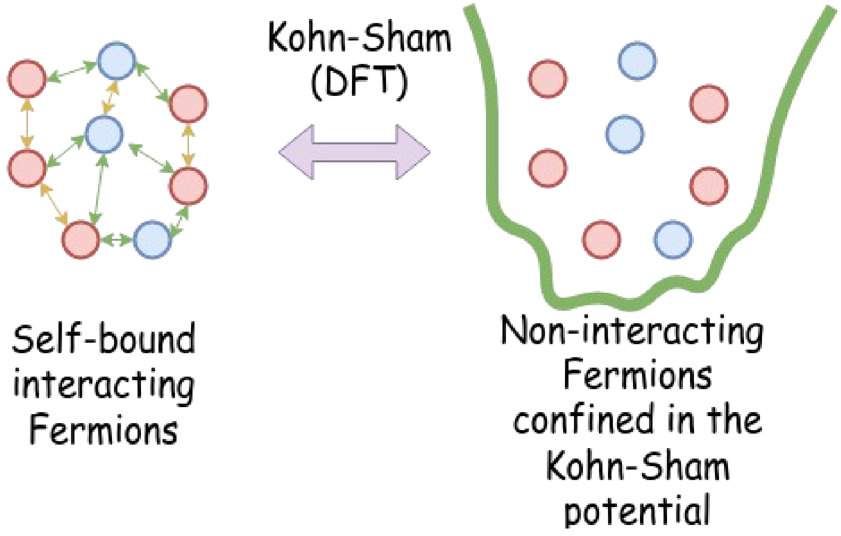
$$\min_{\Psi} \langle \Psi | T + V + V_{\text{ext}} | \Psi \rangle = \min_{\rho} E[\rho]$$

so **E[ρ]** has a **minimum** for the **exact ground-state density** where it assumes the **exact energy** as a value.

Kohn-Sham realization ($F[\rho] \rightarrow T[\rho]$)

For **any interacting system**, there exists a **local single-particle potential $V_{KS}(r)$**, such that the **exact ground-state** density of the **interacting system** equals the **ground-state density** of the **auxiliary non-interacting system**:

$$\rho_{\text{exact}}(\vec{r}) = \rho_{\text{KS}}(\vec{r}) = \sum_{i=1}^{A} |\phi(\vec{r})|^2$$



where φ are single-particle orbitals and the total wave-function correspond to a Slater determinant. The **E[ρ] is unique**

$$E[\rho] = T[\rho] + \int V_{\text{KS}}(\vec{r})\rho(\vec{r})d\vec{r}$$

where **T[ρ]** is the **kinetic energy of the non-interacting system** and for which the variational equation

$$0 = \frac{\delta E[\rho]}{\delta\rho} = \frac{\delta T[\rho]}{\delta\rho} + V_{\text{KS}}$$

yields to the **exact ground state density and energy**

Nuclear Energy Density Functionals

- The **Kohn-Sham potential** V_{KS} can be customarily split in **three pieces**:

$$V_{\mathrm{KS}} = V_{\mathrm{Hartree}} + V_{\mathrm{Exch\&Corr}} + V_{\mathrm{external}}$$

in **nuclei $V_{external}$ = 0** , the **Hartree** contribution to **E[ρ]** is **easy to calculate** once an interaction has been assumed. The **difficulty** is usually to determine the **exchange & correlation** part.

$$E_{\mathrm{Hartree}} = \frac{1}{2}\int d\vec{r} \int d\vec{r}\,'\rho(\vec{r})v(\vec{r}-\vec{r}\,')\rho(\vec{r}\,')$$

- **Kohn-Sham** scheme depends entirely on **whether** accurate approximations for the **exchange & correlation** potential can be found.
- Due to **Vxc**, the KS goes **beyond** a simple **Hartree-Fock** (**V_{HF} = V_H + V_F**) and it has the **advantage of being local** (in space).

State of the art on current nuclear EDFs

→ Mostly **based** on **effective interactions** solved at the Hartree-Fock – **Mean-Field –** level.

→ Neglect **explicit** correlation effects **$V_{xc} = V_F$** → **Correlation effects** included **implicitly** in the **fitting** parameters of the **model.**

→ **Fitted** to experimental data of **ground state** properties in **medium** and **heavy nuclei** as well as **pseudo-data** (nuclear matter properties derived from other experiment/observations/ calculations)

Such an scheme provides a description of **nuclear masses and charge radii** of about **1%** to **0.1%** and the description of **Giant Resonances** and related **sum rules** with an accuracy of a **few %** being the **only available theoretical framework** that can be applied today to most of the **whole nuclear chart**.

There exist **two main successful approaches**:

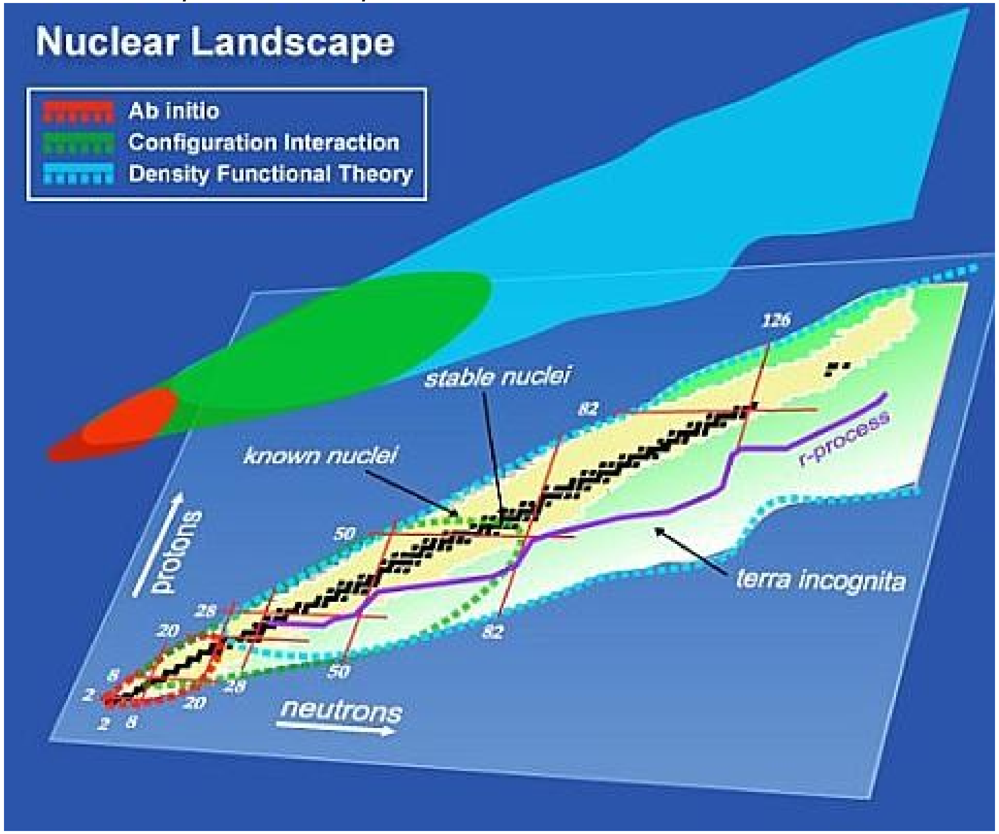
→ Relativistic based on Lagrangians where **effective mesons carry the interaction** (π, σ, ω...).

→ Non-relativistic based on effective Hamiltonians with **Yukawa, Gaussian (e.g. Gogny) or zero-range forces (e.g. Skyrme).**

⇒ Both give **similar results** and in both cases a **density dependence** of the interaction should be incorporated to properly **describe nuclear medium effects (3N, 4N, ...). This clearly corresponds to an EDF philosophy.**

Advantadges and disadvantages of DFT

UNEDF http://unedf.mps.ohio-state.edu/



→ **<u>ADVANTAGES OF DFT:</u>**

- **exact theory** that can be applied to the **whole nuclear chart**
- **many-body** problem mapped onto a **one-body** problem without the need of explicitly involving inter-nucleon interactions!!! (computational cost and interpretation of observables in terms of single-particle properties)
- **HK generalised in (almost all) possible ways**: time dependence, degenerate ground-state, magnetic systems, finite T, relativistic case ...
- **any one body observable is within the DFT framework** (this includes also some sum rules related to nuclear excitations)

→ **<u>DISADVANTAGES OF DFT:</u>**

- various **proofs of HK theorems** do **not** give any clue on **how to build the functional**.
- **no** direct **connection** with **realistic NN or NNN interaction** if current approaches to EDF are not improved (some attempts already exist)
- **no systematic** way of **improvement** (evaluate syst. Errors) so far.

Skyrme type of EDF

Zero-range **Skyrme** interaction is a **low momentum transfer** expansion of a **Yukawa** type of interaction up to O[P²]:

$$
\begin{aligned}
V(\boldsymbol{r}_1,\boldsymbol{r}_2) = \; & t_0\,(1+x_0P_\sigma)\,\delta(\boldsymbol{r}) && \text{central term} \\
& +\frac{1}{2}t_1\,(1+x_1P_\sigma)\left[\boldsymbol{P}'^2\delta(\boldsymbol{r})+\delta(\boldsymbol{r})\boldsymbol{P}^2\right] && \\
& +t_2\,(1+x_2P_\sigma)\,\boldsymbol{P}'\cdot\delta(\boldsymbol{r})\boldsymbol{P} && \text{non-local terms} \\
& +\frac{1}{6}t_3\,(1+x_3P_\sigma)\,[\rho\,(\boldsymbol{R})\,]^\sigma\,\delta(\boldsymbol{r}) && \text{density-dependent term} \\
& +iW_0\boldsymbol{\sigma}\cdot\left[\boldsymbol{P}'\times\delta(\boldsymbol{r})\boldsymbol{P}\right] && \text{spin–orbit term}\,.
\end{aligned}
$$

$\boldsymbol{r}=\boldsymbol{r}_1-\boldsymbol{r}_2\,,\qquad \boldsymbol{R}=\frac{1}{2}(\boldsymbol{r}_1+\boldsymbol{r}_2)\,,$

$\boldsymbol{P}=\frac{1}{2i}(\boldsymbol{\nabla}_1-\boldsymbol{\nabla}_2)\,,\qquad \boldsymbol{P}'$ cc of $\boldsymbol{P}$ acting on the left $\qquad \boldsymbol{\sigma}=\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2\,,\qquad P_\sigma=(1+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\,/2\,.$

It is also useful to **classify** the anti-symmetrized interaction **V_{AS} = V(1-$P_r P_\sigma P_\tau$)** in the different **spin** and **isospin channels**, that is, schematically:

$$V = V_{\rm s-is} + V_{\rm v-is}(\vec{\sigma}\cdot\vec{\sigma}) + V_{\rm s-iv}(\vec{\tau}\cdot\vec{\tau}) + V_{\rm v-iv}(\vec{\sigma}\cdot\vec{\sigma})(\vec{\tau}\cdot\vec{\tau})$$

Allows to **study** each channel by **probing** the nucleus using **different** types of **operators** (O) **→ H'=H+λO** (e.g. in perturbation theory, RPA or TDHF).

Skyrme energy density: E[ρ,τ,Δρ,J]

→ From the interaction, one can calculate the Energy density as <HF|V$_{Skyrme}$|HF>

→ Standard χ^2 fit including more flexibility in **Spin-Orbit** and ensuring phenomenological indications for the values of the parameters in the **spin** and **spin-isospin channel**.

$$\mathcal{H}_0 = \frac{1}{4} t_0 \left[(2 + x_0)\,\rho^2 - (2x_0 + 1)\left(\rho_p^2 + \rho_n^2\right) \right] ,$$

$$\mathcal{H}_3 = \frac{1}{24} t_3 \rho^\sigma \left[(2 + x_3)\,\rho^2 - (2x_3 + 1)\left(\rho_p^2 + \rho_n^2\right) \right] ,$$

$$\mathcal{H}_{\text{eff}} = \frac{1}{8} \left[t_1 (2 + x_1) + t_2 (2 + x_2) \right] \tau\rho + \frac{1}{8} \left[t_2 (2x_2 + 1) - t_1 (2x_1 + 1) \right] \left(\tau_p \rho_p + \tau_n \rho_n \right) ,$$

$$\mathcal{H}_{\text{fin}} = \frac{1}{32} \left[3t_1 (2 + x_1) - t_2 (2 + x_2) \right] (\nabla \rho)^2 - \frac{1}{32} \left[3t_1 (2x_1 + 1) + t_2 (2x_2 + 1) \right] \left[(\nabla \rho_p)^2 + (\nabla \rho_n)^2 \right] ,$$

$$\mathcal{H}_{\text{so}} = \frac{1}{2} W_0 \left[\boldsymbol{J} \cdot \nabla \rho + \boldsymbol{J}_p \cdot \nabla \rho_p + \boldsymbol{J}_n \cdot \nabla \rho_n \right] ,$$

$$\mathcal{H}_{\text{sg}} = -\frac{1}{16} (t_1 x_1 + t_2 x_2)\, \boldsymbol{J}^2 + \frac{1}{16} (t_1 - t_2) \left[\boldsymbol{J}_p^2 + \boldsymbol{J}_n^2 \right] .$$

SAMi functional

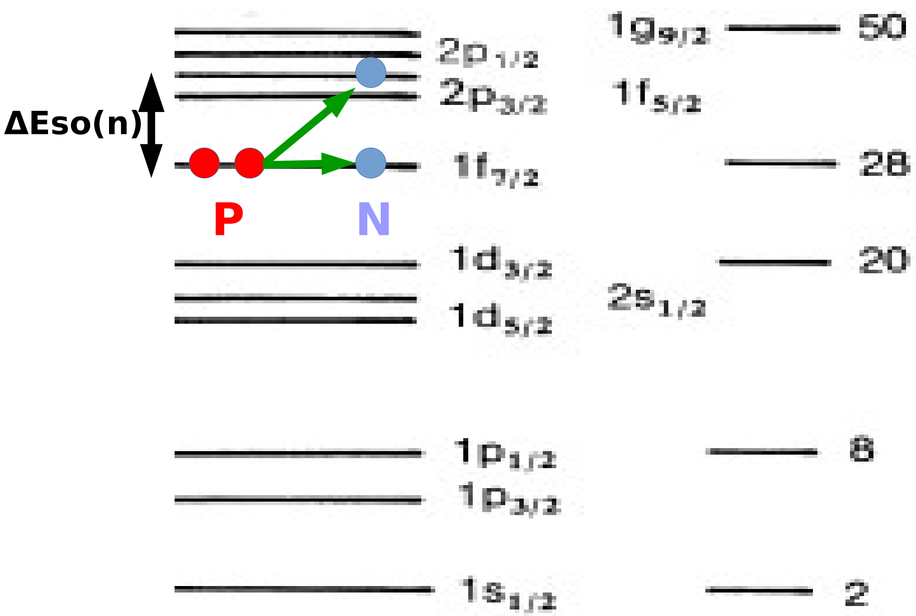
→ **Devised** to improve the description of **spin-isospin resonances** in nuclei (drawback in EDFs).

→ Those resonances can be **excited** by **strong probes** [charge-exhange (CE) reactions] and they can **decay** via the **weak interaction** (axial-vector current couples to the spin and induces β−decay processes).

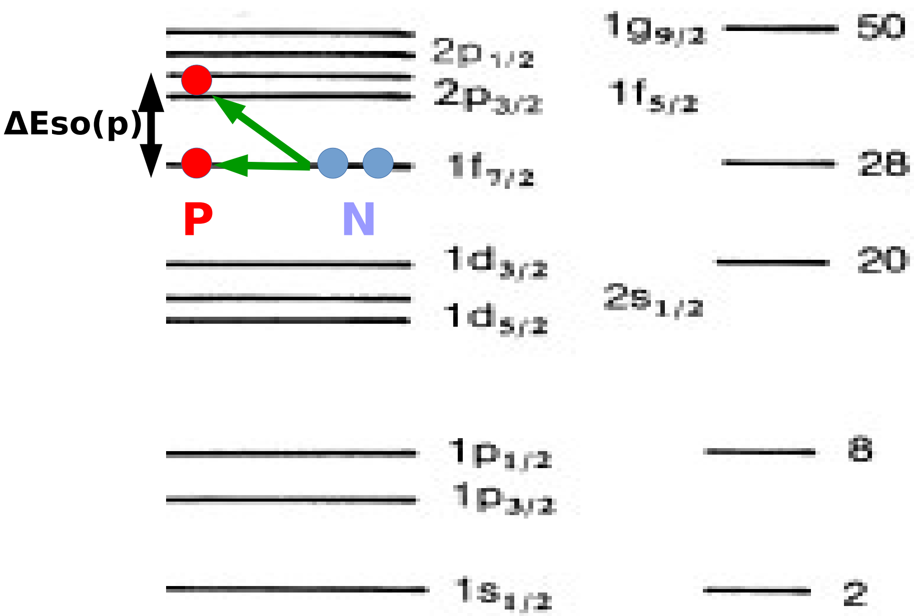
→ **β-decay** and **CE** reaction **mirror transitions** in **isospin** space

→ Hence: allowed **Gamow-Teller** transitions mainly **detemine β-decay half-lives**; **GT** transitions determine **weak interaction rates** essential role in the core-collapse dynamics of massive stars leading to supernova explosion; **GT matrix elements** are **necessary** for the study of **double-β-decay**

Transitions from ^{42}Ti: β-decay

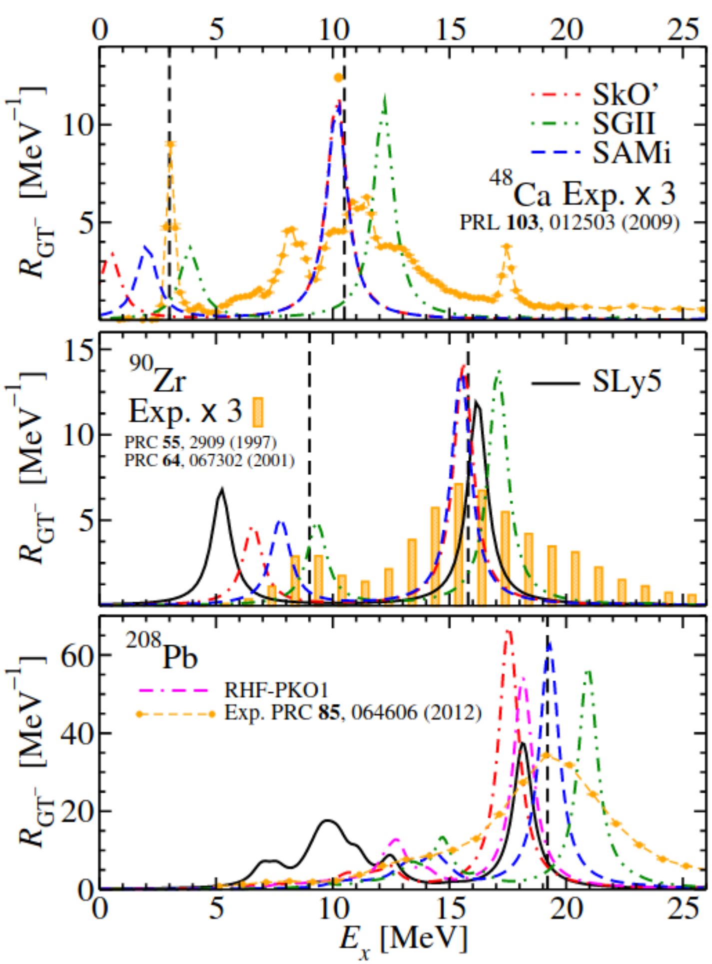


Transitions from ^{42}Ca: Gamow-Teller

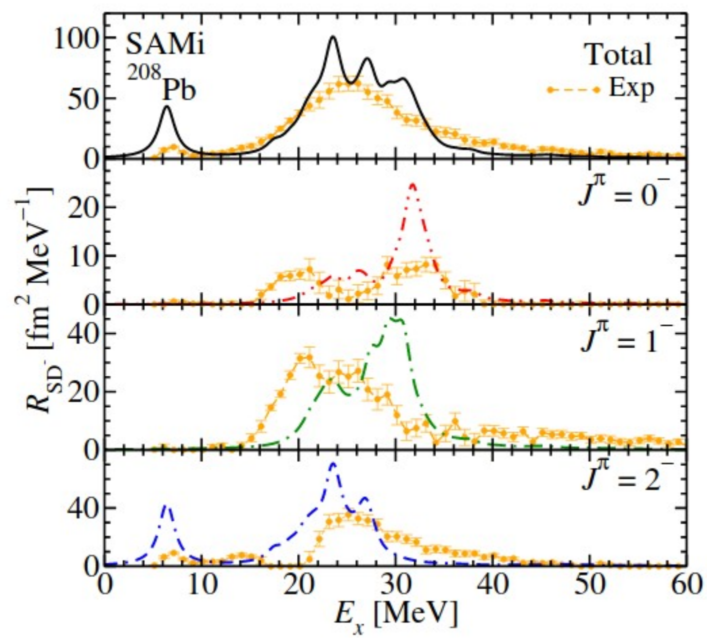


SAMi: results spin-isospin resonances

GAMOW-TELLER [O~στ±]



SPIN-DIPOLE [O~τ± r^L (Y x σ)_L]

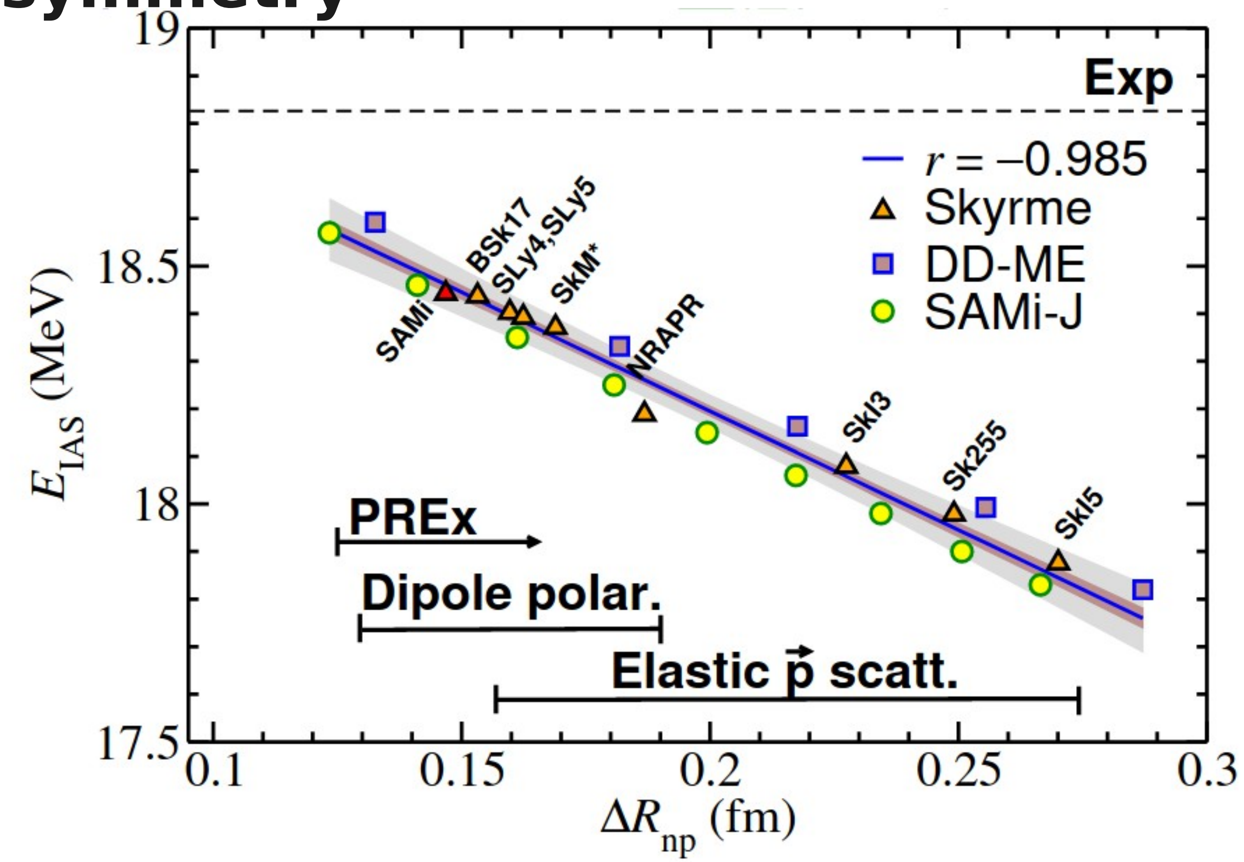


$$\int [R_{SD^-}(E) - R_{SD^+}(E)]dE = \frac{9}{4\pi}\left(N\langle r_n^2\rangle - Z\langle r_p^2\rangle\right)$$
$$\approx (N-Z)\langle r_p^2\rangle \left(1 + \frac{2N}{N-Z}\frac{\Delta r_{np}}{\langle r_p^2\rangle^{1/2}}\right)$$

A new Skyrme interaction with improved spin-isospin properties -X. Roca-Maza, G. Colò, H. Sagawa - Phys.Rev. C86 (2012) 031306.

Isobaric analog resonance (pure isospin resonance O~τ)

Standard nuclear EDFs (such as SAMi): the nuclear part is **isospin symmetric** and only **Coulomb term brakes isospin symmetry**



Exp errors in IAS ~ tens of keV

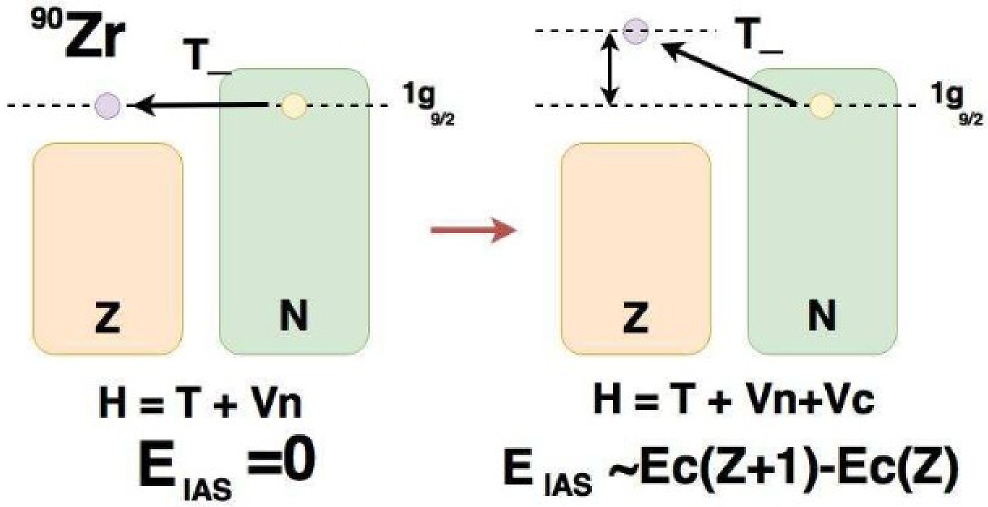
(or smaller)

Exp width IAS

~ hundreds of keV

Nuclear Symmetry Energy and the Breaking of the Isospin Symmetry: How Do They Reconcile with Each Other? - X. Roca-Maza, G. Colò, and H. Sagawa - Phys. Rev. Lett. 120, 202501 (2018)

Isobaric analog resonance Energy: E_{IAS}



- **Analog state** can be defined: $|A\rangle = \frac{T_-|0\rangle}{\langle 0|T_+T_-|0\rangle^{1/2}}$
- **Displacement energy or** E_{IAS}

$$E_{IAS} = E_A - E_0 = \langle A|\mathcal{H}|A\rangle - \langle 0|\mathcal{H}|0\rangle = \frac{\langle 0|T_+[\mathcal{H},T_-]|0\rangle}{\langle 0|T_+T_-|0\rangle}$$

$E_{IAS} \neq 0$ **only due to Isospin Symmtry Breaking terms** $\mathcal{H}$

E_{IAS}^{exp} **usually accuratelly measured !**

Estimates of the possible contributions

$[\mathcal{H}, T_-] \neq 0$? essentially **Coulomb potential** but not only

Table: Estimate of the different effects on E_{IAS} in ^{208}Pb.

	E_{IAS} Correction
Coumb direct	$\sim$ 20 MeV
Coulomb exchange	$\sim$ -300 keV
n-p mass difference	$\sim$ tens keV
Electromagnetic spin-orbit	$\sim$ - tens keV
Finite size effects	$\sim$ - 100 keV
Short range correlations	$\sim$ 100 keV
sospin impurity	$\sim$ -100 keV
Isospin symmetry breaking	$\sim$ - 250 keV
	$\sim$ 19 MeV

Physical Review Letters **23**, 484 (1969).

$E_{IAS}^{exp} = 18.83 \pm 0.01$ MeV. *Nuclear Data Sheets 108, 1583 (2007).*

Coulomb direct contribution (leading term): a simple model

- Assuming indepentent particle model and good isospin for $|0\rangle$ ($\langle 0|T_+T_-|0\rangle = 2T_0 = N - Z$)

$$E_{IAS} \approx E_{IAS}^{C,direct} = \frac{1}{N-Z}\int [\rho_n(\vec{r}) - \rho_p(\vec{r})]\, U_C^{direct}(\vec{r})\, d\vec{r}$$

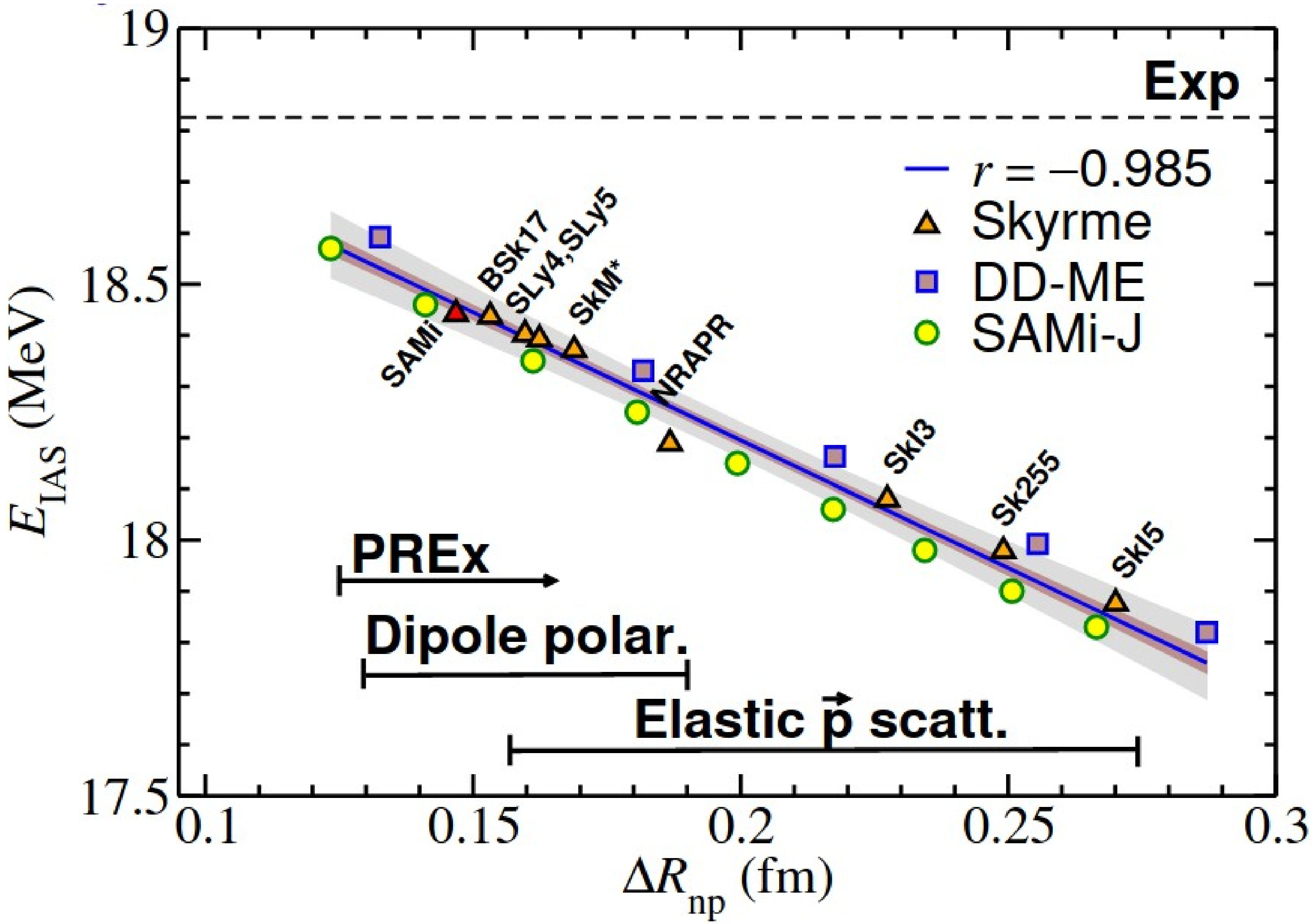
where $U_C^{direct}(\vec{r}) = \int \frac{e^2}{|\vec{r}_1 - \vec{r}|}\rho_{ch}(\vec{r}_1)\, d\vec{r}_1$

- Assuming also a uniform neutron and proton distributions of radius R_n and R_p respectively, and $\rho_{ch} \approx \rho_p$ one can find

$$E_{IAS} \approx E_{IAS}^{C,direct} \approx \frac{6}{5}\frac{Ze^2}{R_p}\left(1 - \sqrt{\frac{5}{12}\frac{N}{N-Z}\frac{\Delta r_{np}}{R_p}}\right)$$

One may expect: **the larger the Δr_{np} the smallest E_{IAS}**

E_{IAS} versus Δr_{np} from EDFs



X. Roca-Maza, G. Colò, and H. Sagawa, Phys. Rev. Lett. 120, 202501 (2018)

Corrections:
implemented within the HF+RPA

Within the **HF+RPA** one can **estimate** the $E_{\rm IAS}$ accounting (in an effective way) for **short-range correlations and effects of the continuum** (if a large sp base is adopted).

- **Coulomb exchange** exact (usually Slater approx.):

$$U_C^{x,\rm exact}\varphi_i(\vec r) = -\frac{e^2}{2}\int d^3r' \,\frac{\varphi_j^*(\vec r\,')\varphi_j(\vec r)}{|\vec r - \vec r\,'|}\varphi_i(\vec r\,')$$

- The **electromagnetic spin-orbit** correction to the nucleon single-particle energy (non-relativistic),

$$\varepsilon_i^{\rm emso} = \frac{\hbar^2c^2}{2m_i^2c^4}\langle \vec l_i\cdot\vec s_i\rangle x_i \int \frac{1}{r}\frac{dU_C}{dr}|R_i(r)|^2$$

where x_i: $g_p - 1$ for Z and g_n for N; $g_n = -3.82608545(90)$ and $g_p = 5.585694702(17)$, $R_i \to R_{nl}$ radial wf.

Corrections:
implemented within the HF+RPA

- **Finite size** effects (assuming spherical symmetry):

$$\begin{aligned}\rho_{ch}(q) &= \left(1 - \frac{q^2}{8m^2}\right)\left[G_{E,p}(q^2)\rho_p(q) + G_{E,n}(q^2)\rho_n(q)\right] \\ &- \frac{\pi q^2}{2m^2}\sum_{l,t}\left[2G_{M,t}(q^2) - G_{E,t}(q^2)\right]\langle\vec{l}\cdot\vec{s}\rangle\int_0^\infty dx\frac{j_1(qx)}{qx}|R_{nl}(x)x^2|^2\end{aligned}$$

- **Vacuum polarization:** lowest order correction in the fine-structure constant to the Coulomb potential $\frac{eZ}{r}$:

$$V_{vp}(\vec{r}) = -\frac{2}{3}\frac{\alpha e^2}{\pi}\int d\vec{r}'\frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|}\mathcal{K}_1\left(\frac{2}{\lambda\!\!\!^{-}_e}|\vec{r}-\vec{r}'|\right)$$

where e is the fundamental electric charge, α the fine-structure constrant, $\lambda\!\!\!^{-}_e$ the reduced Compton electron wavelength and

$$\mathcal{K}_1(x) \equiv \int_1^\infty dt e^{-xt}\left(\frac{1}{t^2} + \frac{1}{2t^4}\right)\sqrt{t^2-1}$$

Corrections:
implemented within the HF+RPA

- Isospin symmetry breaking (Skyrme-like): two parts (contact interaction)

charge symmetry breaking +

$$V_{CSB} = V_{nn} - V_{pp}$$

$V_{CSB}(\vec{r}_1, \vec{r}_2) \equiv \frac{1}{4}[\tau_z(1) + \tau_z(2)]\, s_0 (1 + y_0 P_\sigma)$

τ_z Pauli in isospin space; P_σ are the usual projector operators in spin space.

charge independence breaking*

$$V_{CIB} = \frac{1}{2}(V_{nn} + V_{pp}) - V_{pn}$$

$V_{CIB}(\vec{r}_1, \vec{r}_2) \equiv \frac{1}{2}\tau_z(1)\tau_z(2)\, u_0 (1 + z_0 P_\sigma)$

* general operator form $\tau_z(1)\tau_z(2) - \frac{1}{3}\vec{\tau}(1)\cdot\vec{\tau}(2)$.

Our prescription $\tau_z(1)\tau_z(2)$ not change structure of HF+RPA.

- Opposite to the other corrections, **ISB contributions depends on new parameters that need to be determined!**

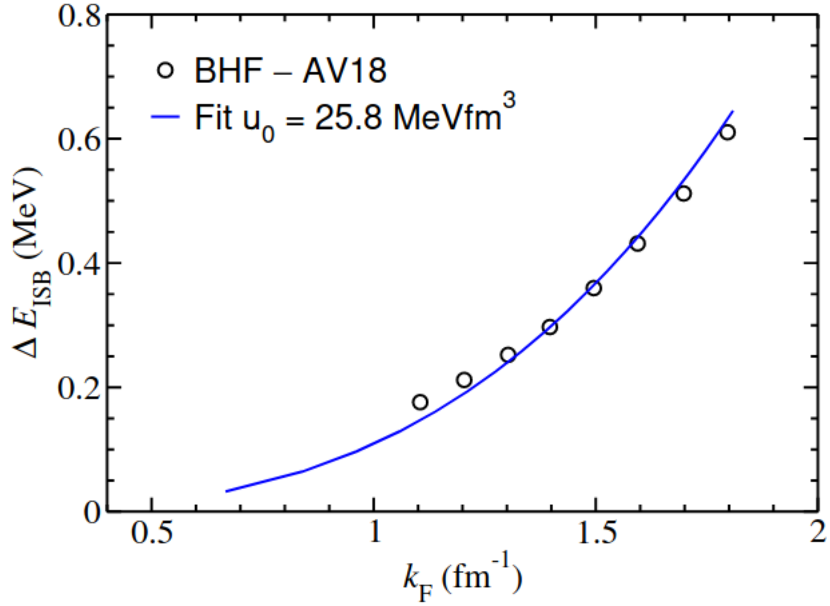
Isospin symmetry breaking in the nuclear medium?

- **keeping** things **simple**: **CSB and CIB** interaction just **delta function** depending on s_0 **and** u_0. **Different possibilities:**

→ **Fitting** to (two) experimentally known **IAS energies**

→ **Derive from theory**

→ **our option:** u_0 **to reproduce BHF** (symmetric nuclear matter) and s_0 **to reproduce** E_{IAS} **in** ^{208}**Pb**



Physics Letters B 445, 259 (1999)

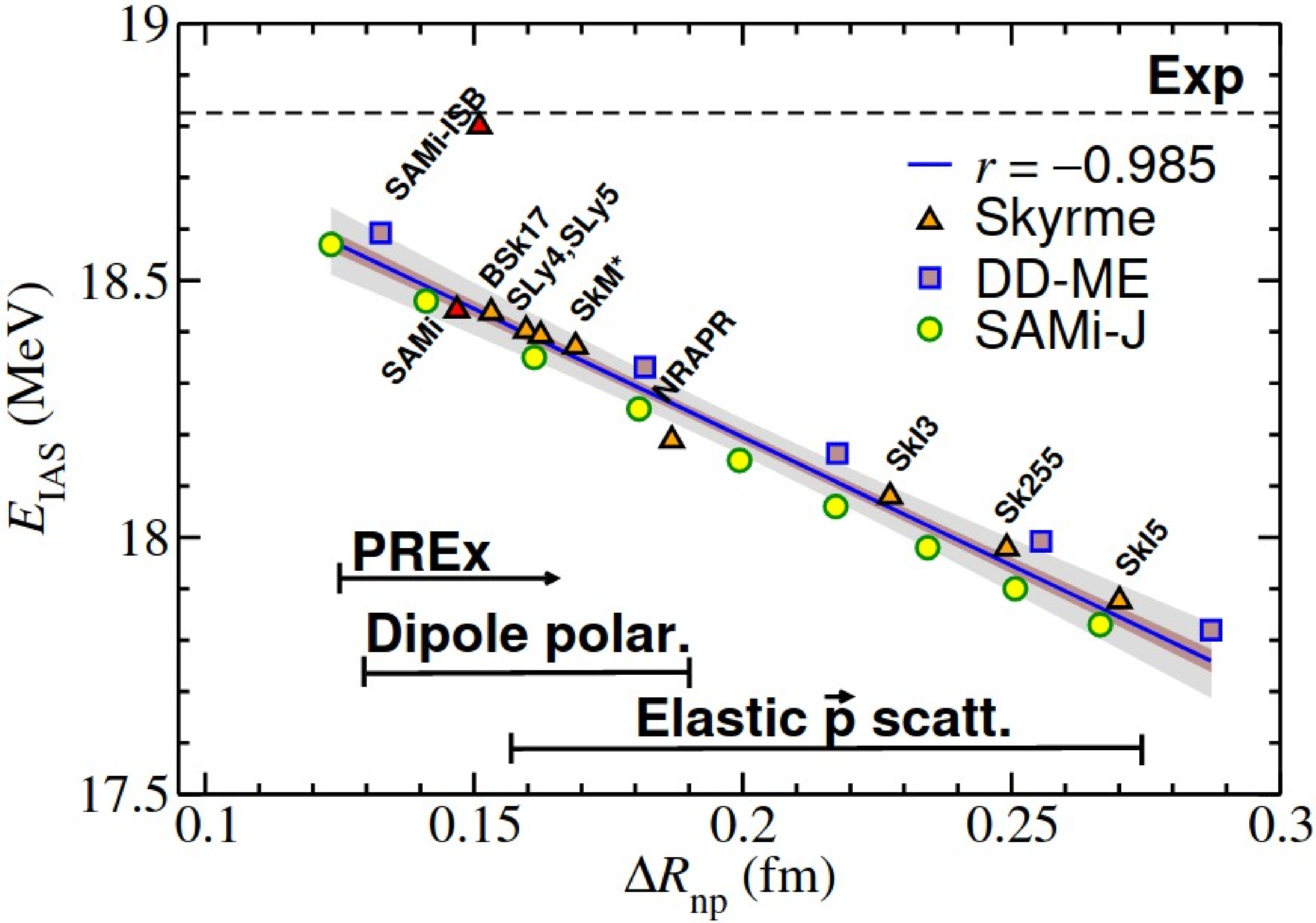
Re-fit of SAMi: SAMi-ISB

- All these **corrections** are relatively **small** but **modify binding energies, neutron and proton distributions, etc.**
$\Rightarrow$ a **re-fit of the interaction is needed**.
- Use **SAMi fitting protocol** (special care for spin-isospin resonances) including all corrections and **find SAMi-ISB**

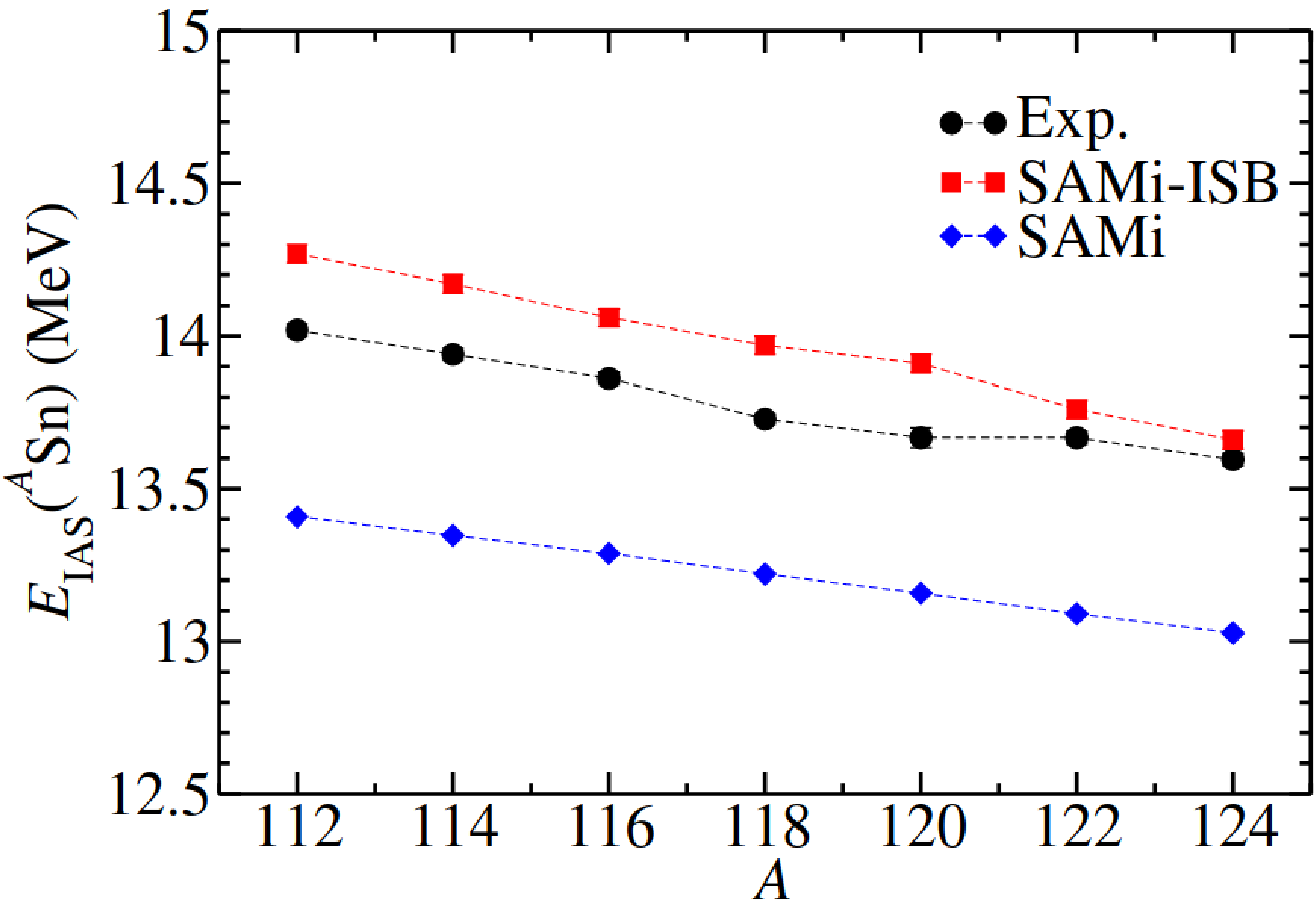
Table: Saturation properties

	SAMi	SAMi-ISB	
ρ_∞	0.159(1)	0.1613(6)	fm^{-3}
e_∞	−15.93(9)	−16.03(2)	MeV
m^*_{IS}	0.6752(3)	0.730(19)	
m^*_{IV}	0.664(13)	0.667(120)	
J	28(1)	30.8(4)	MeV
L	44(7)	50(4)	MeV
K_∞	245(1)	235(4)	MeV

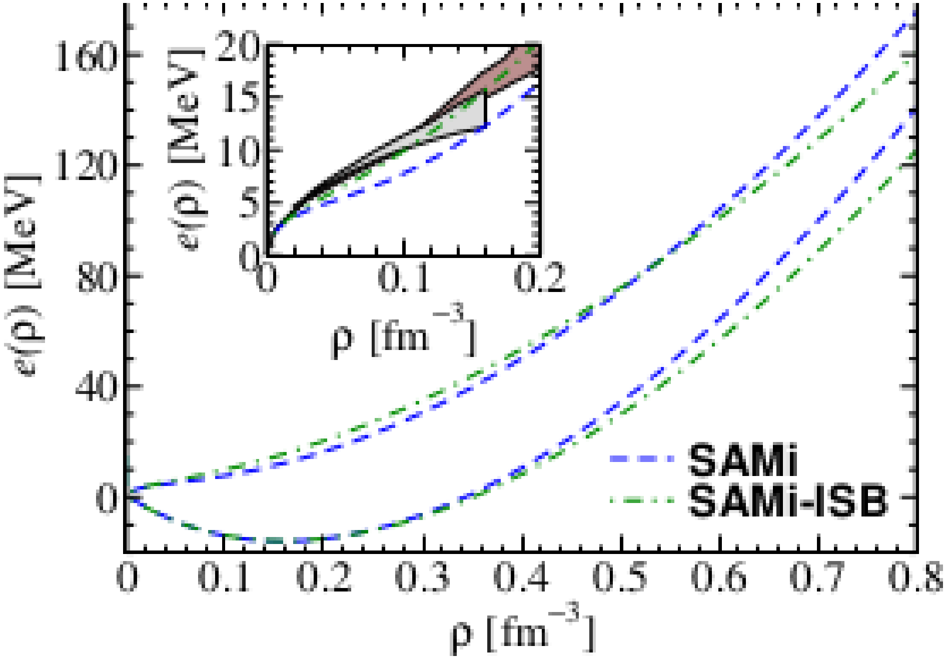
E_{IAS} with SAMi-ISB

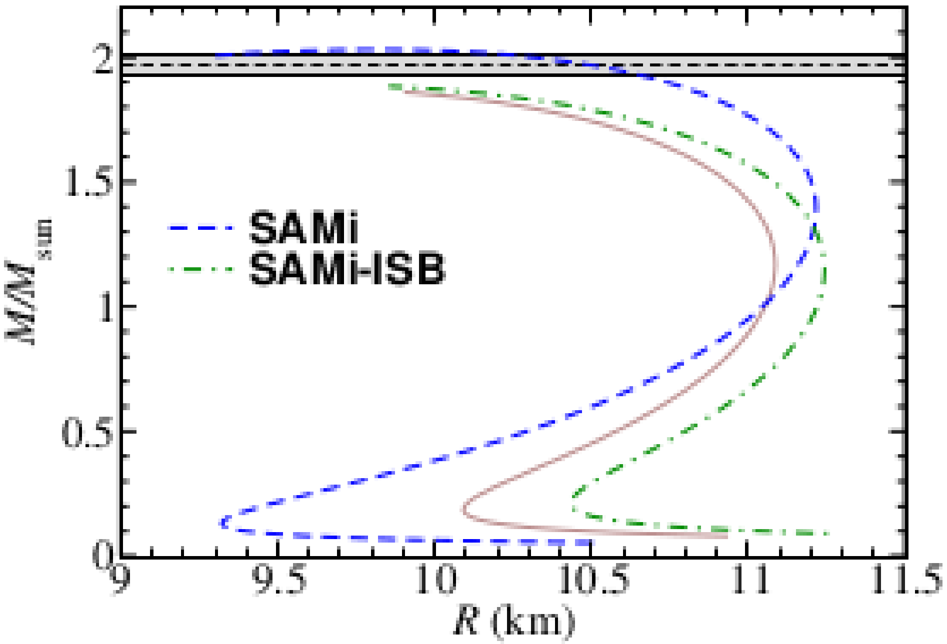


Prediction: E_{IAS} in the Sn isotopic chain



SAMi & SAMi-ISB predictions for NS





	M_{max}/M_{sun}	R_{max} [km]	$\rho^c_{1.4}$ [fm^{-3}]	$R_{1.4}$ [km]	$\Lambda_{1.4}$	$\xi_{1.4}$
SAMi	2.03	9.8	0.5385	11.22	301	0.1845
SAMi-ISB	1.88	9.8	0.5938	11.16	261	0.1853
SAMi-ISB ($u_0 = s_0 = 0$)	1.86	9.9	0.6105	11.01	242	0.1879

Based on **Master Thesis work** of Giovanni Selva

Conclusions:

→ We have **successfully** determined new Skyrme EDF which accounts for the **most relevant quantities** in order to improve the description of **charge-exchange** nuclear **resonances**:

→ the phenomenological indications for the strength of the **spin** and **spin-isospin** channels of the **interaction**.

→ the proton **spin-orbit** splittings of different high angular momenta single-particle levels

→ The **GTR** and **SDR** are reasonably **well described** by **SAMi** and **SAMi-ISB**

→ **No EDF** is compatible with experimental data and **IAR** data at the same time **unless ISB terms are included** (SAMi-ISB)

→ **SAMi functionals** do not deteriorate the **description** of other nuclear observables such as **masses, radii, and non-charge exchange resonances**

→ applicability in **nuclear physics and astrophysics**

Avenues to improve EDFs?

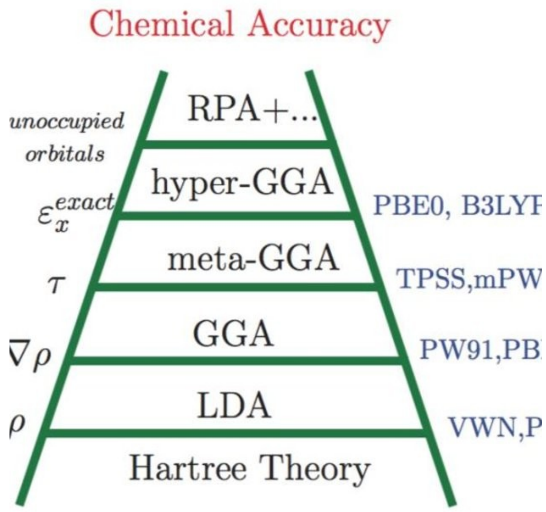
→ We are working in two main directions:

1) **Inverse Kohn-Sham problem**: determine the V_{KS} and then E[ρ,...] from experimental and/or ab initio density distributions. With different **Bachelor and Master Thesis** students

First step in the nuclear inverse Kohn-Sham problem: From densities to potentials

G. Accorto, P. Brandolini, F. Marino, A. Porro, A. Scalesi, G. Colò, X. Roca-Maza, and E. Vigezzi
Phys. Rev. C **101**, 024315 – Published 28 February 2020

2) **Mimic** strategy (**Jacob's Ladder**) in **many-electron systems** to systematically improve nuclear EDFs without using *phenomenological* parameters (as long as possible). With one **PhD** (Francesco Marino) and hopefully one postdoc in the future.



Testing and understanding many-body theories

→ **Harmonic potential theorem:** should be fulfilled by any meaningful many-body theory. With **Bachelor** student and **PostDoc**.

Harmonic Potential Theorem: Extension to Spin-, Velocity-, and Density-Dependent Interactions

S. Zanoli, X. Roca-Maza, G. Colò, and Shihang Shen (申时行)

Phys. Rev. Lett. **123**, 112501 – Published 13 September 2019

→ **Exactly solvable models:** help in better understanding the involved approximations. With **Master** student and **PostDoc**.

ACCEPTED MANUSCRIPT

Extended lipkin-meshkov-glick hamiltonian

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Accepted Manuscript online 5 January 2021 • © 2020 IOP Publishing Ltd

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[for more details see academic supervision in http://www0.mi.infn.it/~jroca/experience.html]

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