How does subatomic matter organize itself?

A low-energy nuclear physics perspective

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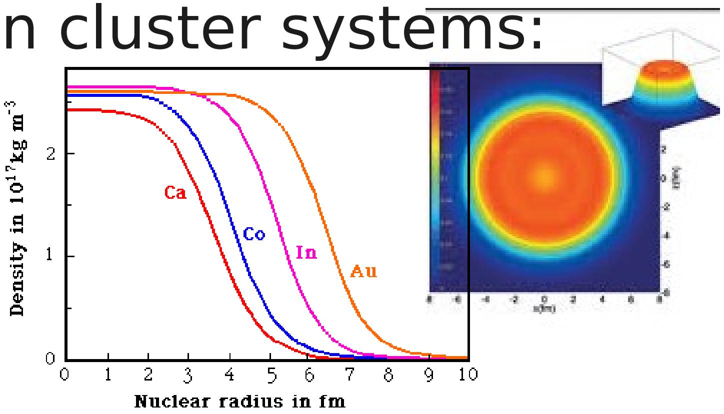
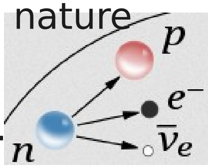
RCNP - August 3rd-5th

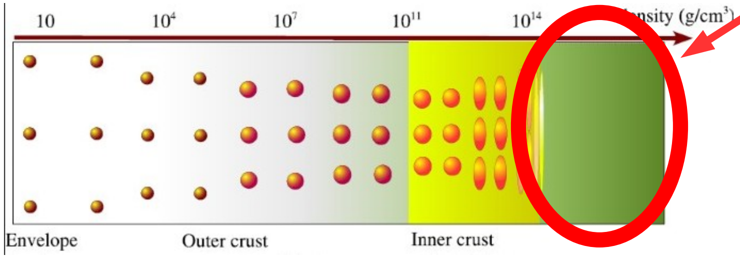
Where can we find neutrons and protons? And in which form? Free? In clusters?

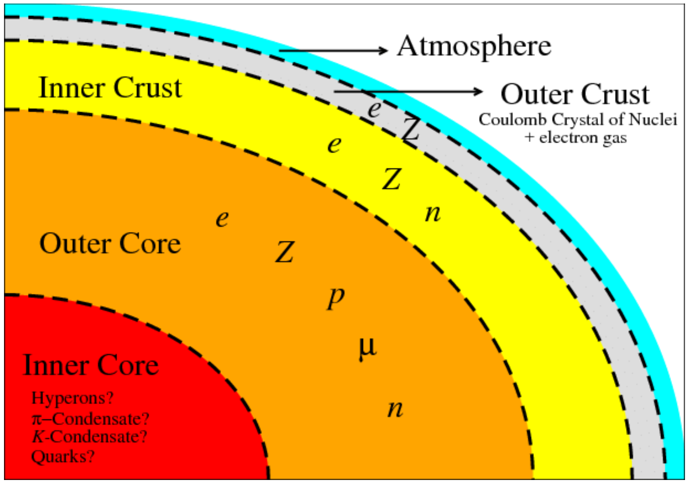
- Neutrons and protons in **Earth** are found in cluster systems: **nuclei**
  - → The **interior** of all nuclei has **constant density** (10^{14} times denser than water) named **saturation density**
  - → Saturation is originated from the **short range** nature of the **nuclear effective interaction**
  - → Neutron in 15 minutes must find a proton or …





- In **heavens**, neutrons and protons can be also found as an interacting Fermi liquid: **matter in the outer core of a neutron star**
  - → Densities can reach several times nuclear saturation

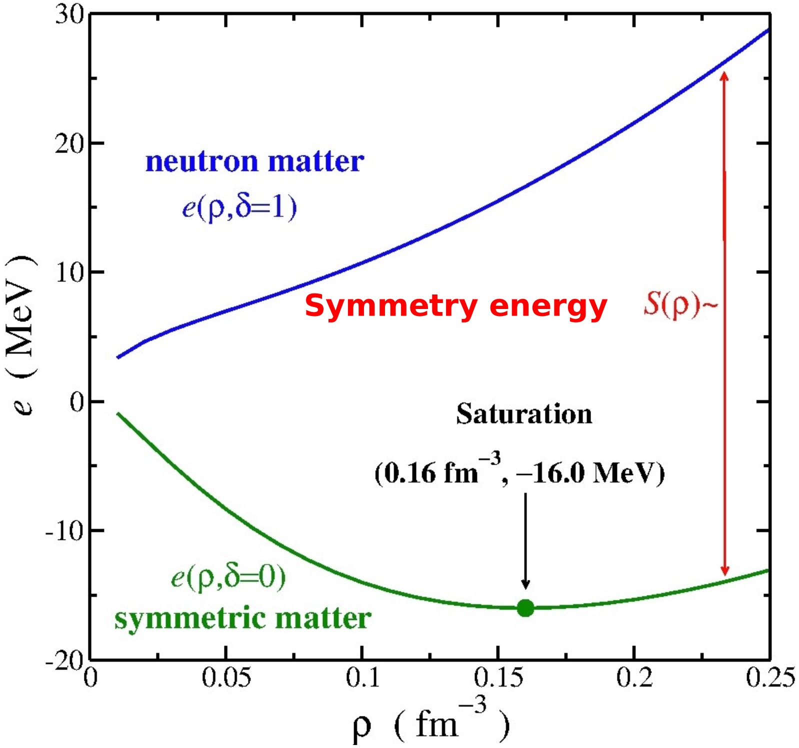




Nuclear Equation of State (EoS)

Unpolarized **nuclear matter** at zero temperature (10^{10}K→1MeV) is defined as the **energy** per **nucleon** (*e*) as a function of the **neutron** (ρ_n) and **proton** (ρ_p) **densities** as (*isospin conserving* $V_{nn} = V_{pp} = V_{np}$):

$$e(\rho,\delta) = e(\rho,0) + S(\rho)\delta^2 + \mathcal{O}[\delta^4] \quad \text{where } \rho = \rho_n + \rho_p \text{ and } \delta = \frac{\rho_n - \rho_p}{\rho}$$



It is customary to **expand *e*(*ρ*,*δ*)** around nuclear **saturation density** $\rho_0 \sim 0.16$ fm^{-3}

$$e(\rho,0) = e(\rho_0,0) + \frac{1}{2}K_0 x^2 + \mathcal{O}[\rho^3] \text{ where } x = \frac{\rho - \rho_0}{3\rho_0}$$

$$S(\rho) = J + Lx + \frac{1}{2}K_{\text{sym}} x^2 + \mathcal{O}[\rho^3, \delta^4]$$

K_0 → how **compressible** is symmetric matter at ρ_0

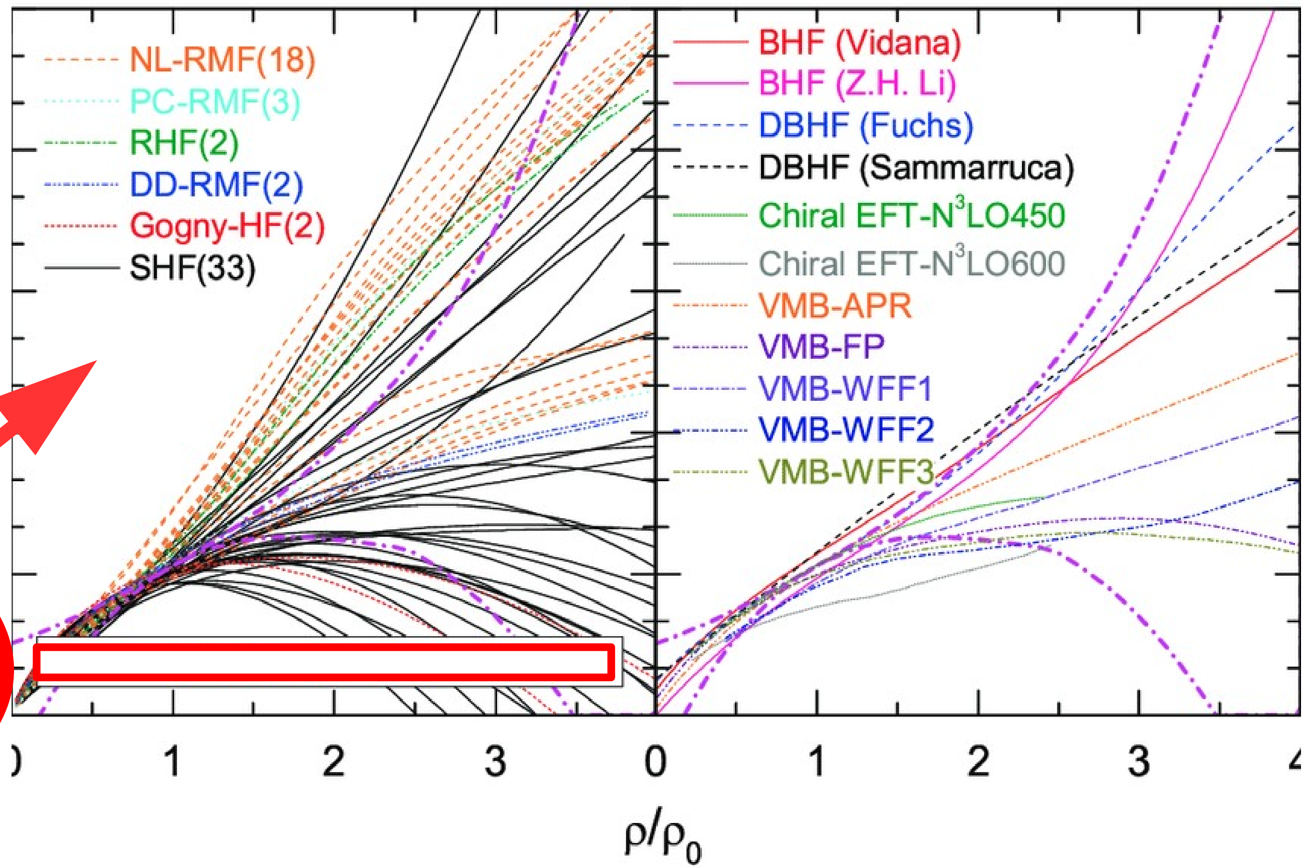
J → **penalty energy** for converting all **protons into neutrons** in symmetric matter at ρ_0

L → **neutron pressure** in neutron matter at ρ_0

Symmetry energy $S(\rho)\sim e(\rho,\delta=1)-e(\rho,\delta=0)$

Symmetry energy not well constrained

Phenomeno-logical models



Microscopic models

Towards understanding astrophysical effects of nuclear symmetry energy
Bao-An Li, Plamen G. Krastev, De-Hua Wen & Nai-Bo Zhang EPJ A 55, 117 (2019)

What can we learn from the Earth and the Heavens about the Nuclear Equation of State?

(some examples)

From Heaven: Neutron Star Mass

Nuclear models that account for different nuclear properties on **Earth** predict a large **variety** of **Neutron Star Mass-Radius** relations →
Observation of a 2Msun has constrained nuclear models.

Tolman–Oppenheimer–Volkoff equation (sph. sym.):

$$\frac{dM(r)}{dr} = 4\pi r^2 \mathcal{E}(r);$$
$$\frac{dP}{dr} = -G\frac{\mathcal{E}(r)M(r)}{r^2}\left[1+\frac{P(r)}{\mathcal{E}(r)}\right]\left[1+\frac{4\pi r^3 P(r)}{M(r)}\right]\left[1-\frac{2GM(r)}{r}\right]^{-1}$$

$\mathcal{E}(r) \rightarrow$ degeneracy pressure from neutrons $\rightarrow M_{\max} = 0.7 M_{\text{sun}}$

Nuclear Physics input is fundamental

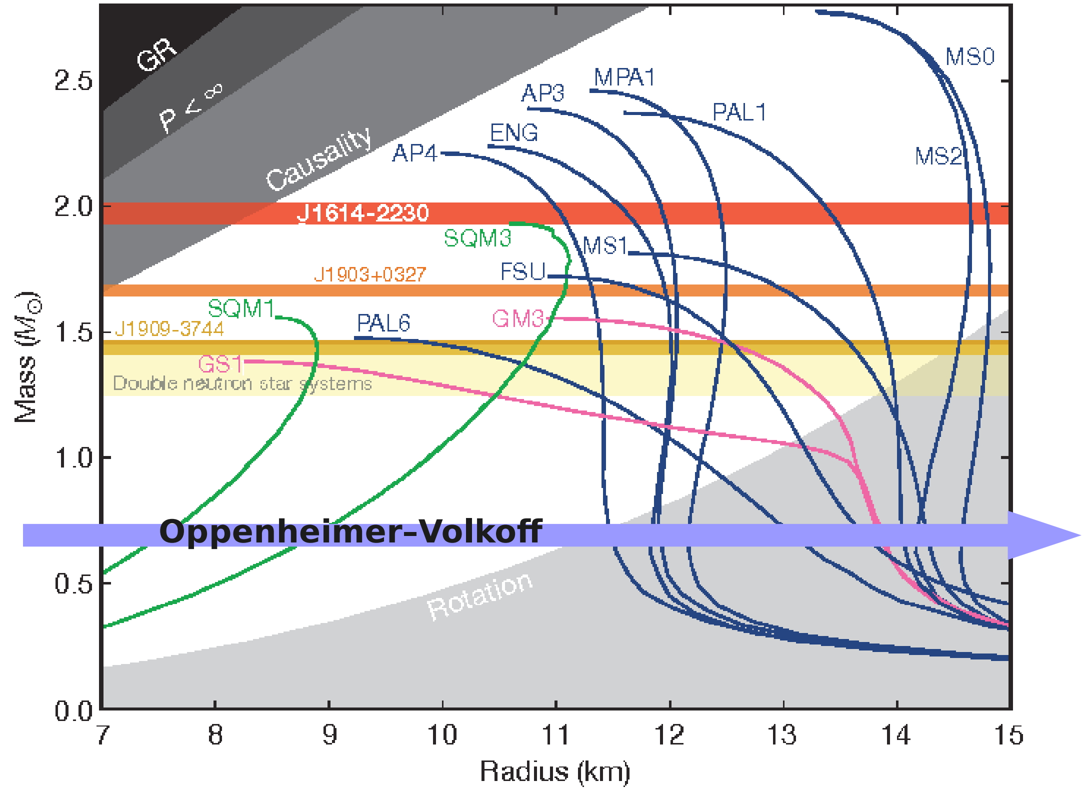
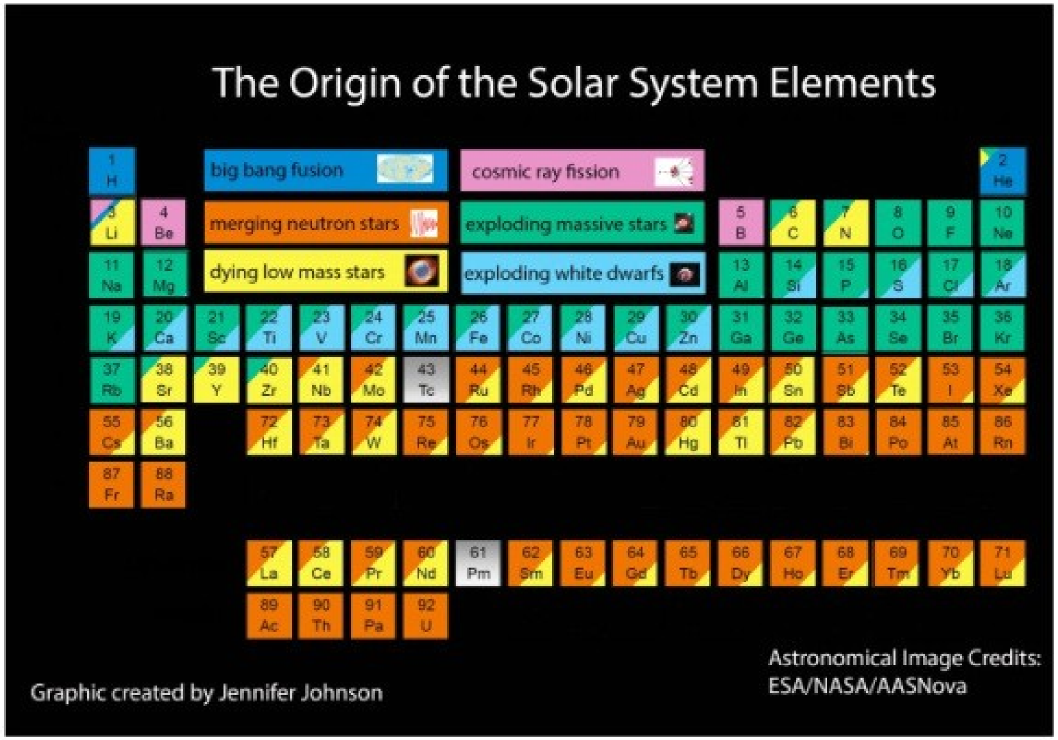


Figure 3 | Neutron star mass–radius diagram. The plot shows non-rotating

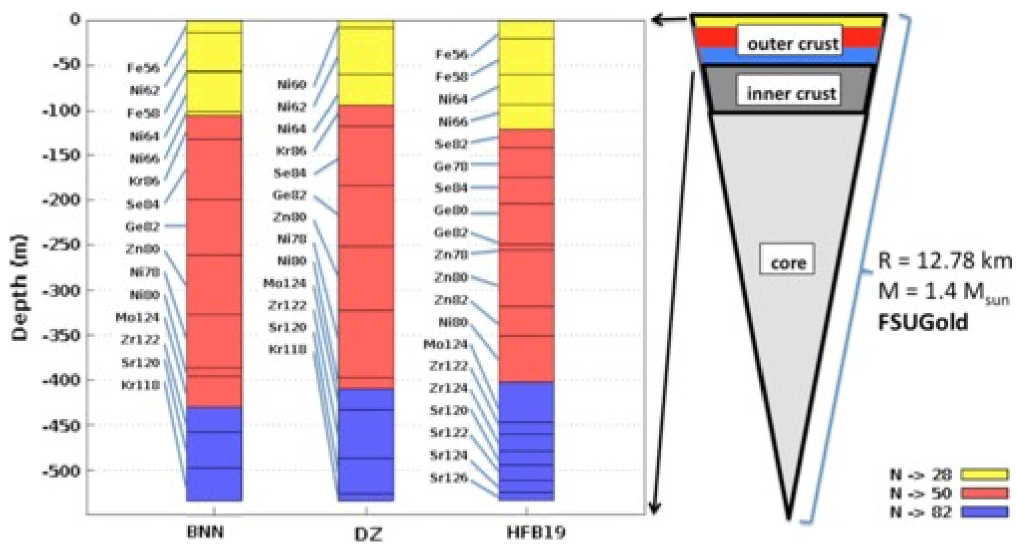
A two-solar-mass neutron star measured using Shapiro delay - P. B. Demorest, T. Pennucci, S. M. Ransom, M. S. E. Roberts & J. W. T. Hessels - Nature volume 467, 1081–1083(2010)

From Heaven: Origin of elements



Binary neutron star merger produced about **10^{29}kg** of **heavy elements**!

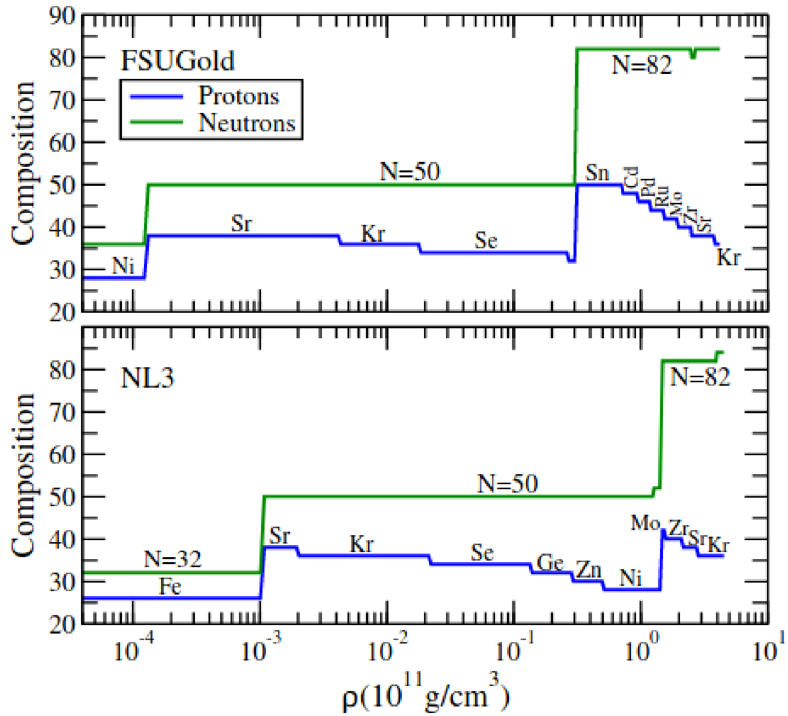
The **crust** of a **NS** is made of very **exotic neutron rich nuclei,** stable only due to the extreme conditions (large densities). **Different nuclear models predict different compositions**



Nuclear mass predictions for the crustal composition of neutron stars: A Bayesian neural network approach R. Utama, J. Piekarewicz, and H. B. Prosper, Phys. Rev. C 93, 014311 (2016)

From Heaven: outer crust composition

- → span 7 orders of magnitude in **denisty** (from **ionization** $\sim 10^4$ g cm to the **neutron drip** $\sim 10^{11}$ g cm)
- → it is organized into a **Coulomb lattice** of neutron-rich nuclei (ions) embedded in a relativistic **uniform electron gas**
- → $T \sim 10^6$ K ~ 0.1 keV → one can treat **nuclei and electrons at $T = 0$ K**
- → At the **lowest densities**, the electronic contribution is negligible so the Coulomb lattice is populated by ^{56}**Fe** nuclei.
- → As the **density increases**, the electronic contribution becomes important, it is energetically advantageous to lower its electron fraction by $e^- + (N, Z) \to (N+1, Z-1) + \nu_e$ and therefore $Z \downarrow$ with constant (approx) number of N
- → As the **density continues to increase, penalty energy from the symmetry energy** due to the neutron excess changes the composition to a dif ferent $N-$*plateau* $\frac{Z}{A} \approx \frac{Z_0}{A_0} - \frac{p_{Fe}}{8a_{sym}}$ where $(A_0, Z_0) = {}^{56}Fe_{26}$
- → The Coulomb lattice is made of more and more neutron-rich nuclei until the critical **neutron-drip density is reached ($\mu_{drip} = m_n$).** $[M(N,Z) + m_n < M(N+1, Z)]$

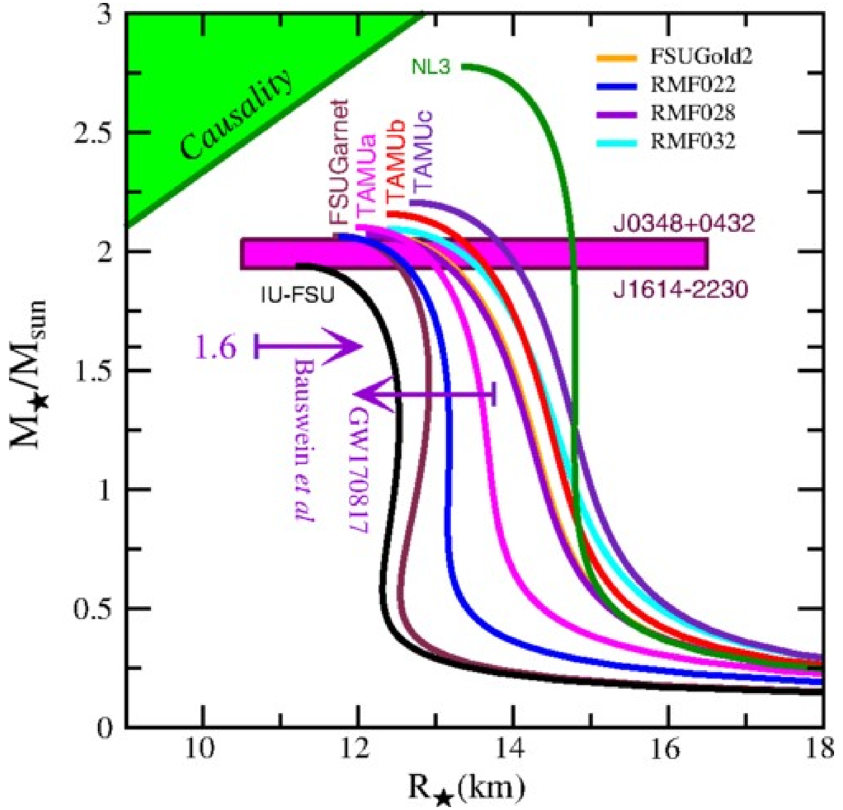


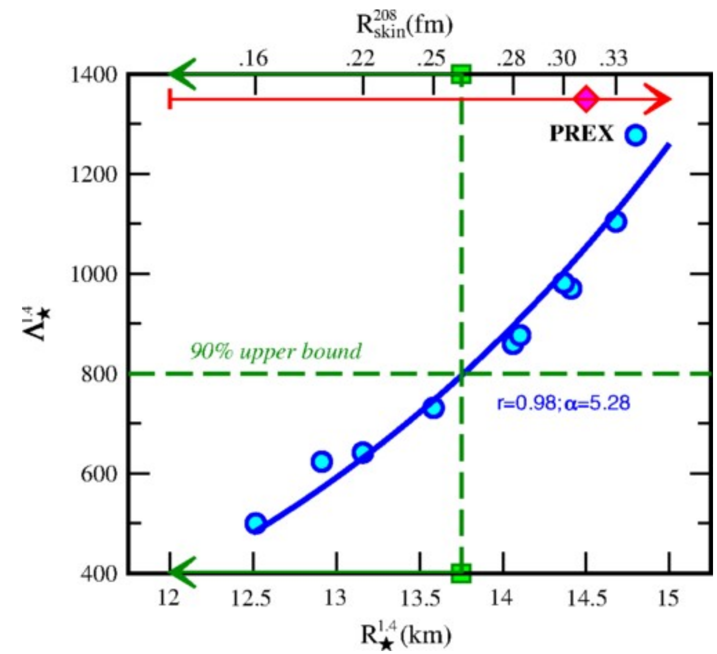
Physical Review C **78**, *025807 (2008)*

The faster the symmetry energy increases with density (L ↑), the more exotic the composition of the outer crust.

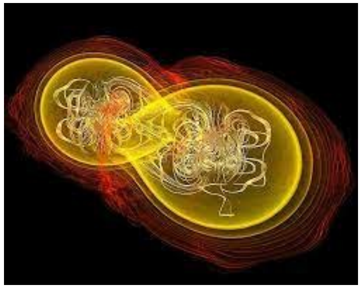
From Heaven: Gravitational wave signal from a binary neutron star merger

GW170817 from the binary neutron star merger → **constraint** neutron star **radius** and, thus, the **nuclear EoS**





Tidal deformability (Λ) is a quadrupole deformation inferred from **GW signal →** proportional to **restoring force.** Hence, sensitive to the **nuclear EoS**



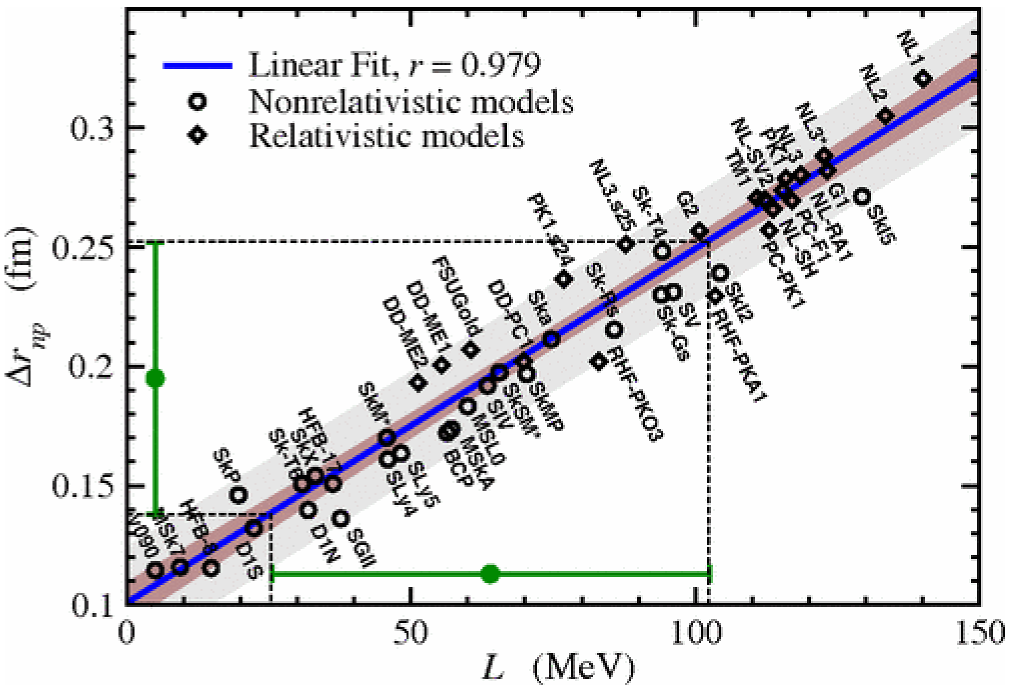
Neutron Skins and Neutron Stars in the Multimessenger Era
F. J. Fattoyev, J. Piekarewicz, and C. J. Horowitz Phys. Rev. Lett. 120, 172702 (2018)

From Heaven & Earth: neutron skin and the Radius of a Neutron Star

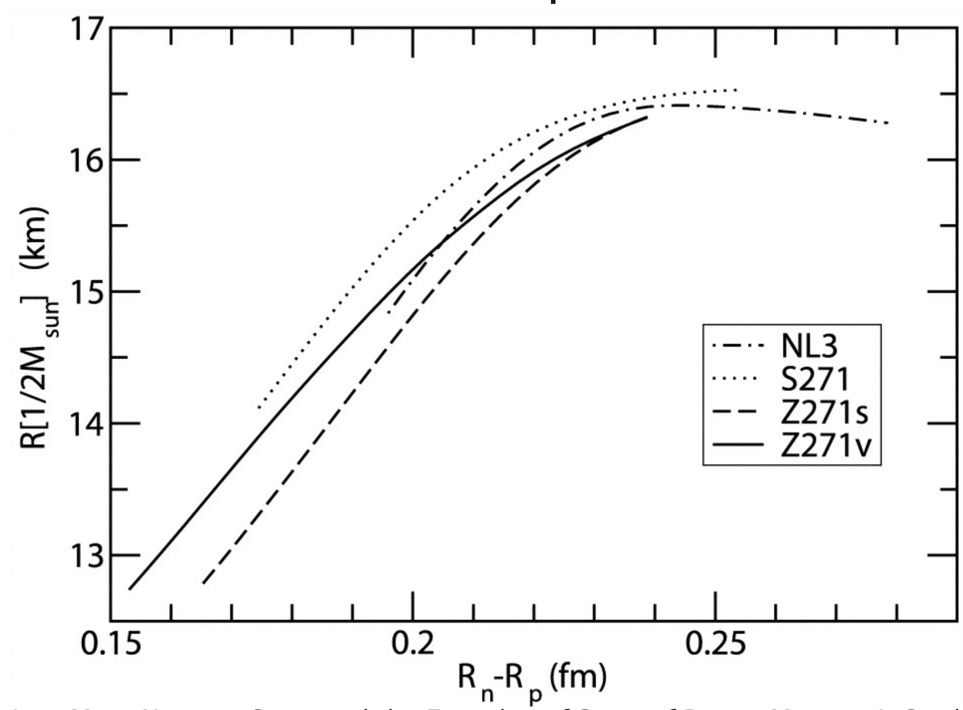
Both, the **neutron skin thickness** ($\Delta r_{np}=r_n-r_p$) in neutron rich nuclei and the **radius** of a **neutron star** are related to the **neutron pressure** in infinite matter. The former around ρ_0 (L) while the latter in a broad range of densities.

$$\Delta r_{np} \approx \frac{1}{12}\frac{N-Z}{A}\frac{R}{J}L$$

→ For **small neutron stars**, that is, for small central densities: nuclear models predict a **linear** relation between **R** and **Δr_{np}**



Neutron Skin of 208Pb, Nuclear Symmetry Energy, and the Parity Radius Experiment
X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda Phys. Rev. Lett. 106, 252501 (2011)



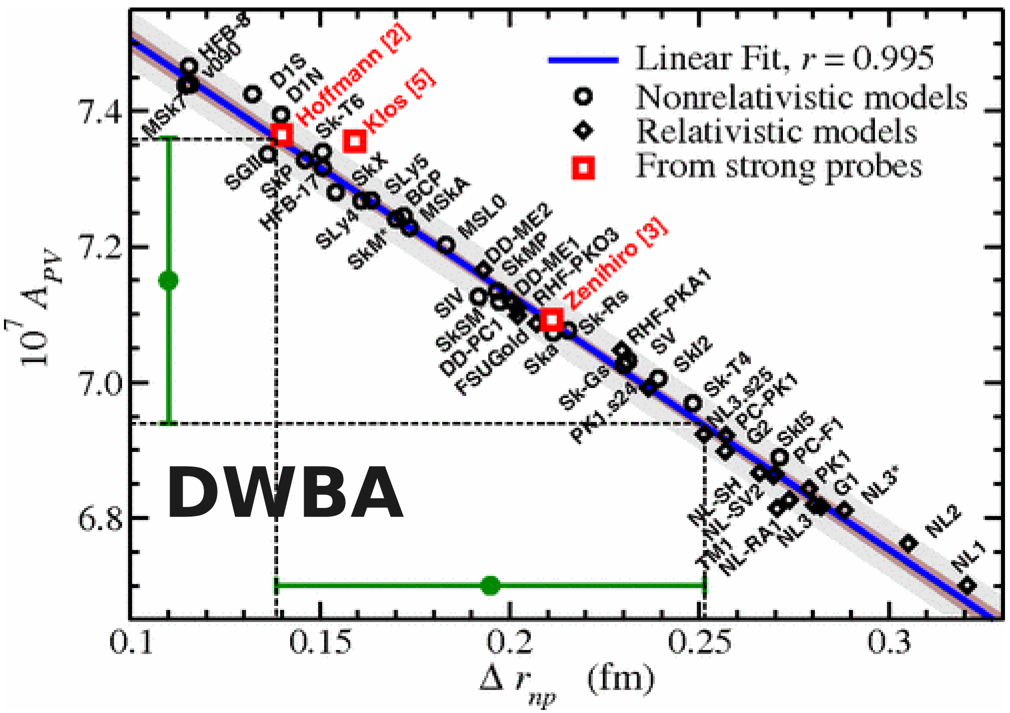
Low-Mass Neutron Stars and the Equation of State of Dense Matter - J. Carriere, C. J. Horowitz, and J. Piekarewicz - The Astrophysical Journal, 593 (2003) 463

From Earth: Parity violating electron scattering and the neutron skin

Polarized electron-Nucleus scattering:

→ In good approximation, **the weak interaction** probes the **neutron distribution** in nuclei while **Coulomb interaction** probes the **proton distribution**

→ **Different experimental efforts @ Jlab (USA) & MAMI (Germany)**



Neutron Skin of 208Pb, Nuclear Symmetry Energy, and the Parity Radius Experiment
X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda Phys. Rev. Lett. 106, 252501 (2011)

→ **Electrons** interact by **exchanging** a **γ** (couples to **p**) or a **Z_0** boson (couples to **n**)

→ **Ultra-relativistic electrons**,depending on their helicity (±), will interact with the nucleus seeing a slightly different potential: **Coulomb** ± **Weak**

$$A_{pv} = \frac{d\sigma_+/d\Omega - d\sigma_-/d\Omega}{d\sigma_+/d\Omega + d\sigma_-/d\Omega} \sim \frac{\text{Weak}}{\text{Coulomb}}$$

→ Main **unknown** is **ρ_n**

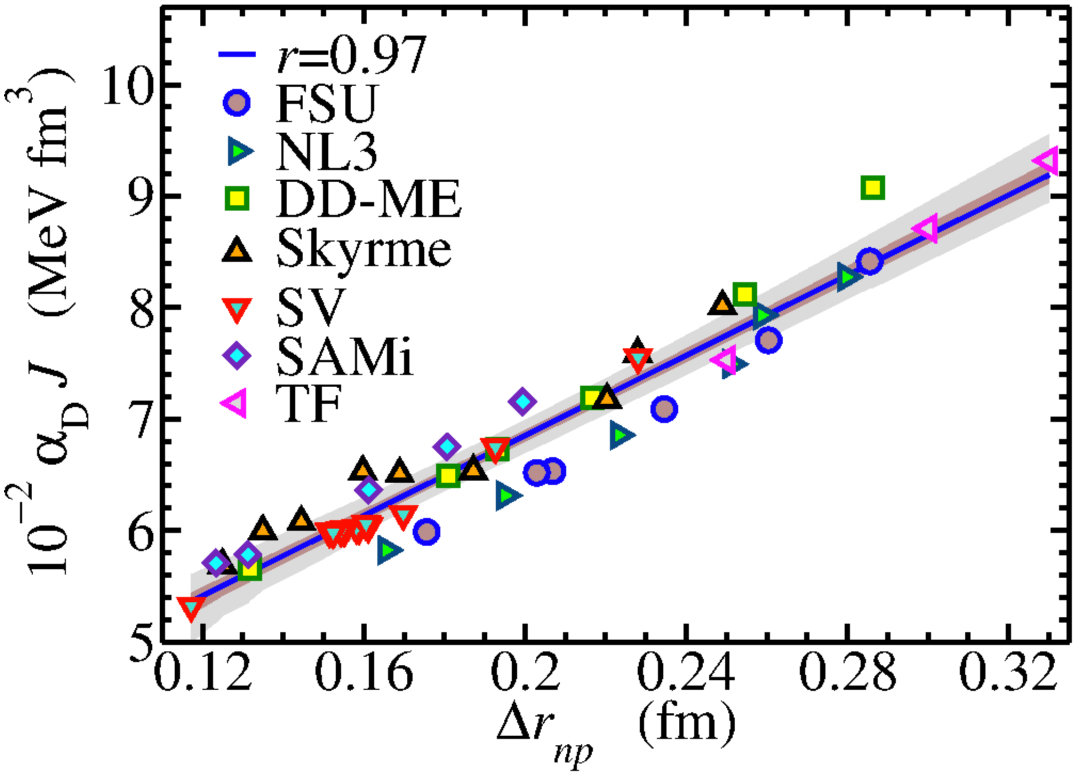
→ In **PWBA** for small momentum transfer **q**:

$$A_{pv} = \frac{G_F q^2}{4\sqrt{2}\pi\alpha}\left(1 - \frac{q^2 r_p^2}{3F_p(q)}\right)\Delta r_{np}$$

From Earth: dipole polarizability and neutron skin

The dipole **polarizability** measures the **tendency** of the nuclear **charge** distribution to be **distorted.**

From a macroscopic point of view **α ~ (electric dipole moment)/(external electric field)**



→ Using the **dielectric theorem**: the polarizability can be computed from the expectation value of the Hamiltonian in the constrained ground state H′=H+λD

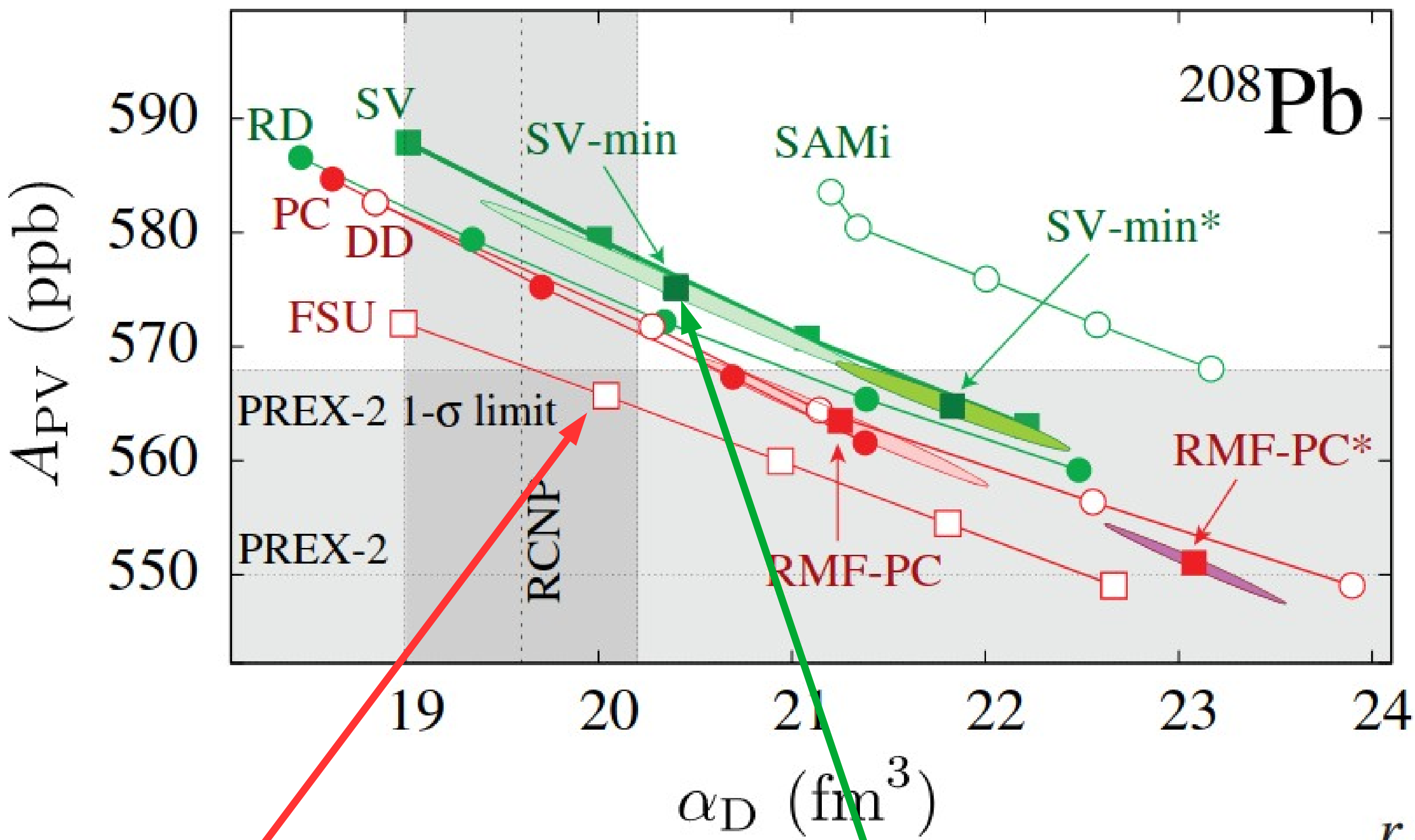
→ For guidance assuming the **Droplet model** for H, one would find:

$$\alpha_D \approx \frac{\pi e^2}{54}\frac{\langle r^2\rangle}{J}A\left(1+\frac{5}{2}\frac{\Delta r_{np}-\Delta r_{np}^{\rm surf}-\Delta r_{np}^{\rm Coul}}{\langle r^2\rangle^{1/2}(I-I_{\rm Coul})}\right)$$

Electric dipole polarizability in 208Pb: Insights from the droplet model - X. Roca-Maza, M. Brenna, G. Colò, M. Centelles, X. Viñas, B. K. Agrawal, N. Paar, D. Vretenar, and J. Piekarewicz

Phys. Rev. C 88, 024316 (2013)

From Earth: A_{pv} (JLab) versus α_D (RCNP)



Paul-Gerhard Reinhard, Xavier Roca-Maza, and Witold Nazarewicz
Phys. Rev. Lett. 127, 232501 (2021)

SV-min, suitable model to predict EoS around ρ_0 and Δr_{np}

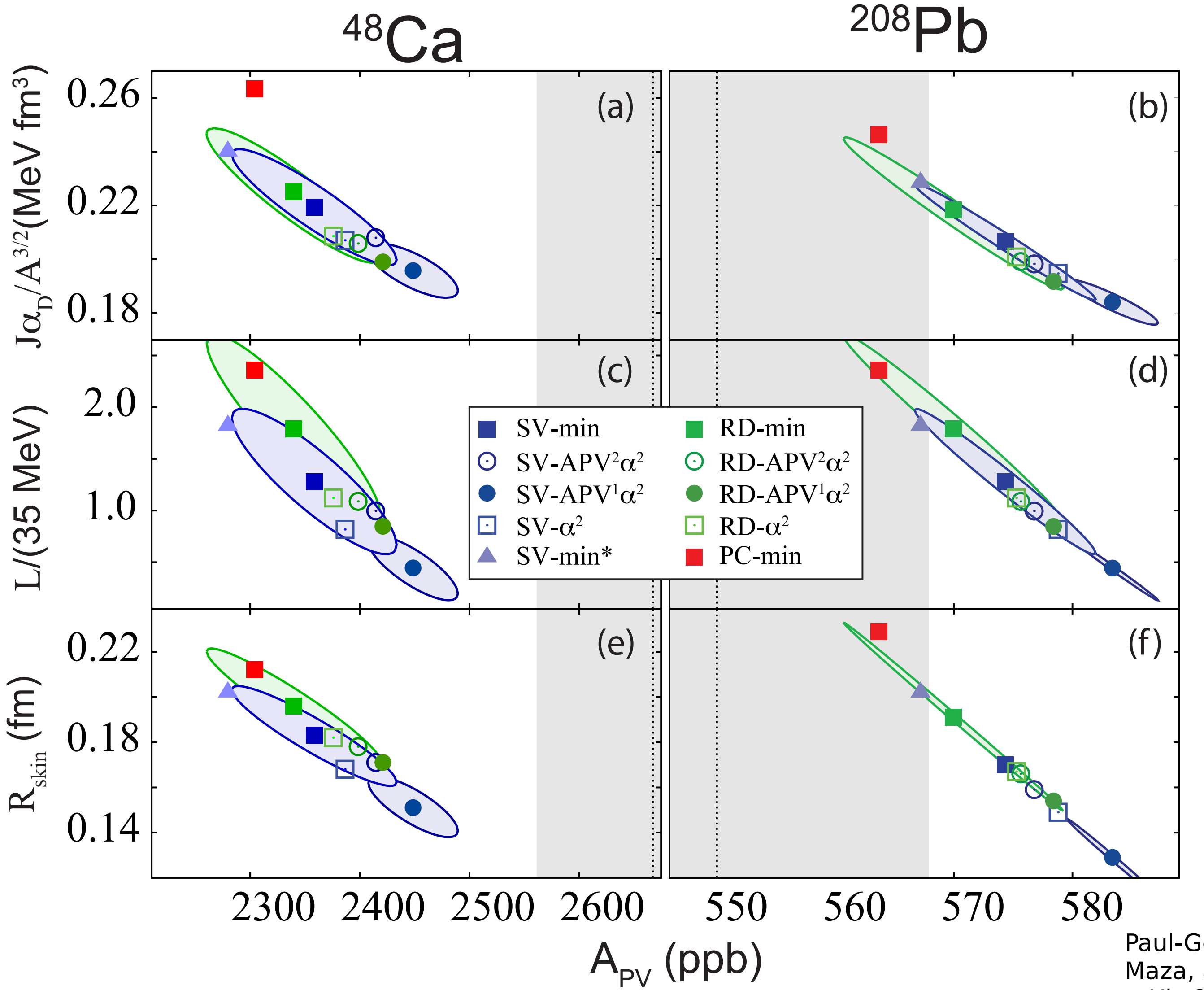
$$r_{skin} = 0.19 \pm 0.02 \text{ fm}$$

$$L = 54 \pm 8 \text{ MeV}$$

B and **Rch** away from "expected" EDF accuracy for B and Rch

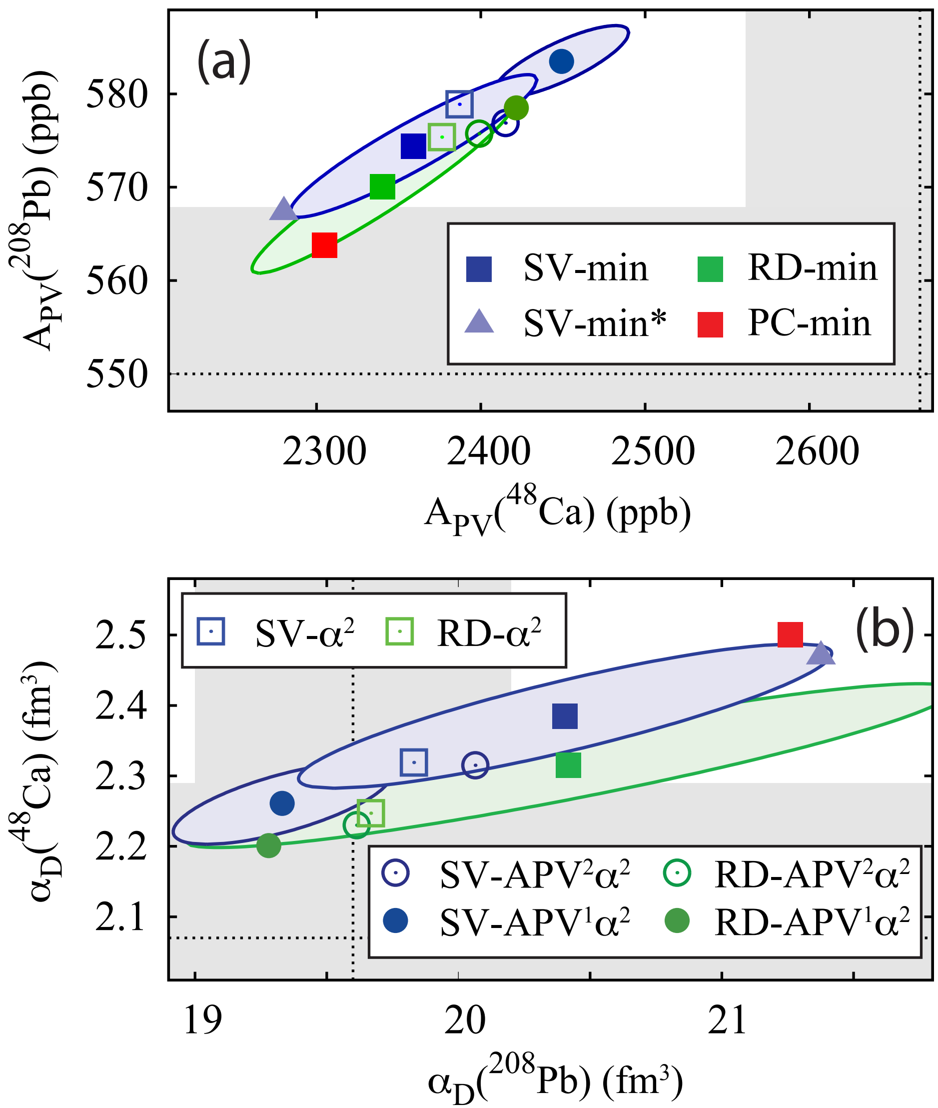
Theoretical and experimental 1σ errors overlap for SV-min
But not overlap both exp.

From Earth: A_{pv} (JLab) versus α_D (RCNP)



Paul-Gerhard Reinhard, Xavier Roca-Maza, and Witold Nazarewicz arXiv:2206.03134

From Earth: A_{PV} (JLab) versus α_D (RCNP)



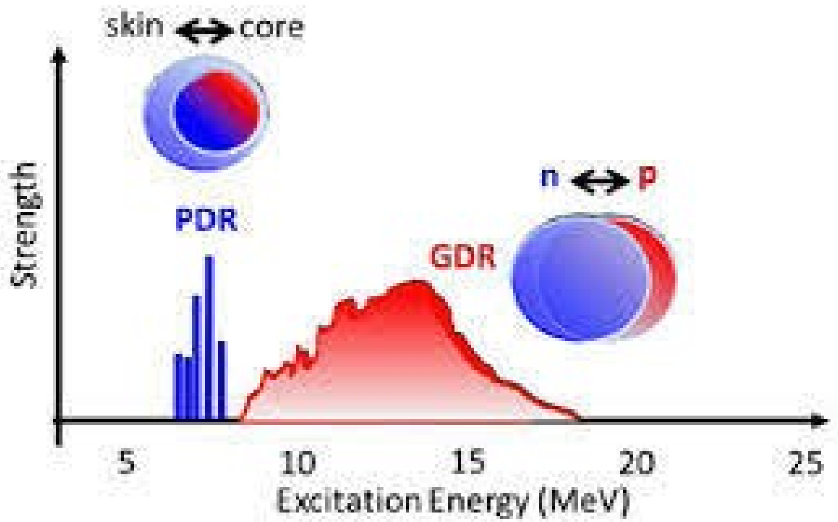
Paul-Gerhard Reinhard, Xavier Roca-Maza, and Witold Nazarewicz
arXiv:2206.03134

From Heaven & Earth: low energy dipole response and nucleosynthesis

The **largest** the **neutron pressure** among neutrons (~L), the more the **excess neutrons** (~skin) are ***"pushed out"*** in the **outermost** part of the **nucleus →** spatial ***decorrelation*** of some of those neutrons with the nucleons in the core produces **larger low lying** responses**.**

GDR=Giant Dipole Resonance

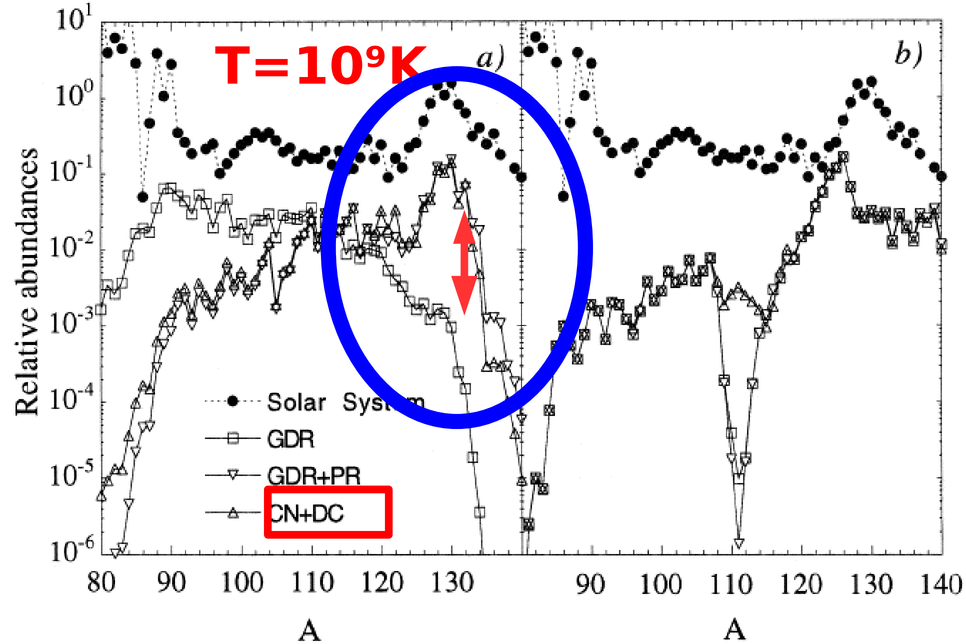
PDR= Pygmy Dipole Resonance



(nature of PDR still under debate)

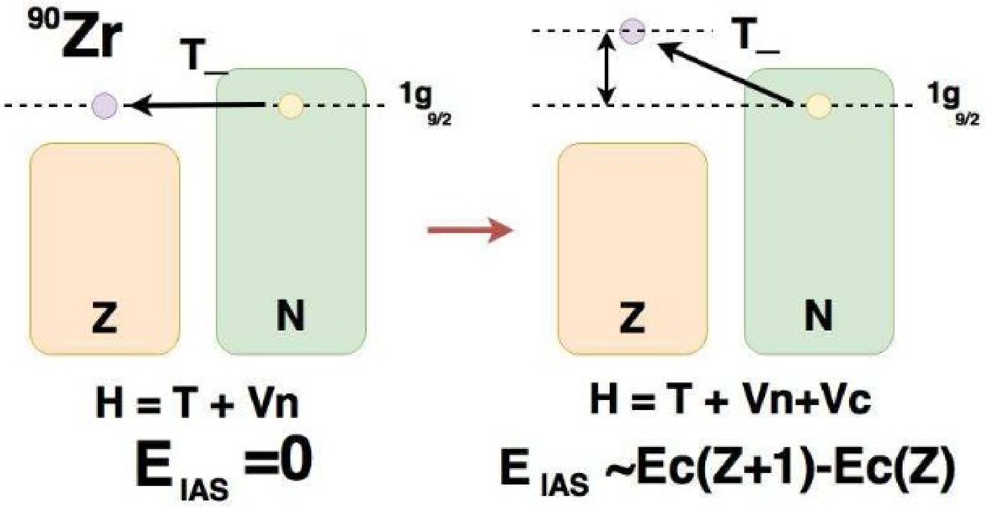
Radiative neutron captures by neutron-rich nuclei and the r-process nucleosynthesis

S. Goriely, Phys.Lett.B 436 (1998) 10-18



Low energy dipole strength in neutron-rich nuclei influences the neutron capture cross section and, thus, the r-process nucleosynthesis

From Earth: Isobaric Analog State and the breaking of isospin symmetry



- **Analog state** can be defined: $|A\rangle = \frac{T_-|0\rangle}{\langle 0|T_+T_-|0\rangle}$
- **Displacement energy or** E_{IAS}

$$E_{IAS} = E_A - E_0 = \langle A|\mathcal{H}|A\rangle - \langle 0|\mathcal{H}|0\rangle = \frac{\langle 0|T_+[\mathcal{H},T_-]|0\rangle}{\langle 0|T_+T_-|0\rangle}$$

$E_{IAS} \neq 0$ **only due to Isospin Symmtry Breaking terms** $\mathcal{H}$

E_{IAS}^{exp} **usually accuratelly measured !**

From Earth: Isobaric Analog State and the breaking of isospin symmetry

→ Coulomb direct contribution: a simple model

- Assuming indepentent particle model and good isospin for $|0\rangle$ ($\langle 0|T_+T_-|0\rangle = 2T_0 = N - Z$)

$$E_{IAS} \approx E_{IAS}^{C,direct} = \frac{1}{N-Z} \int [\rho_n(\vec{r}) - \rho_p(\vec{r})]\, U_C^{direct}(\vec{r})\, d\vec{r}$$

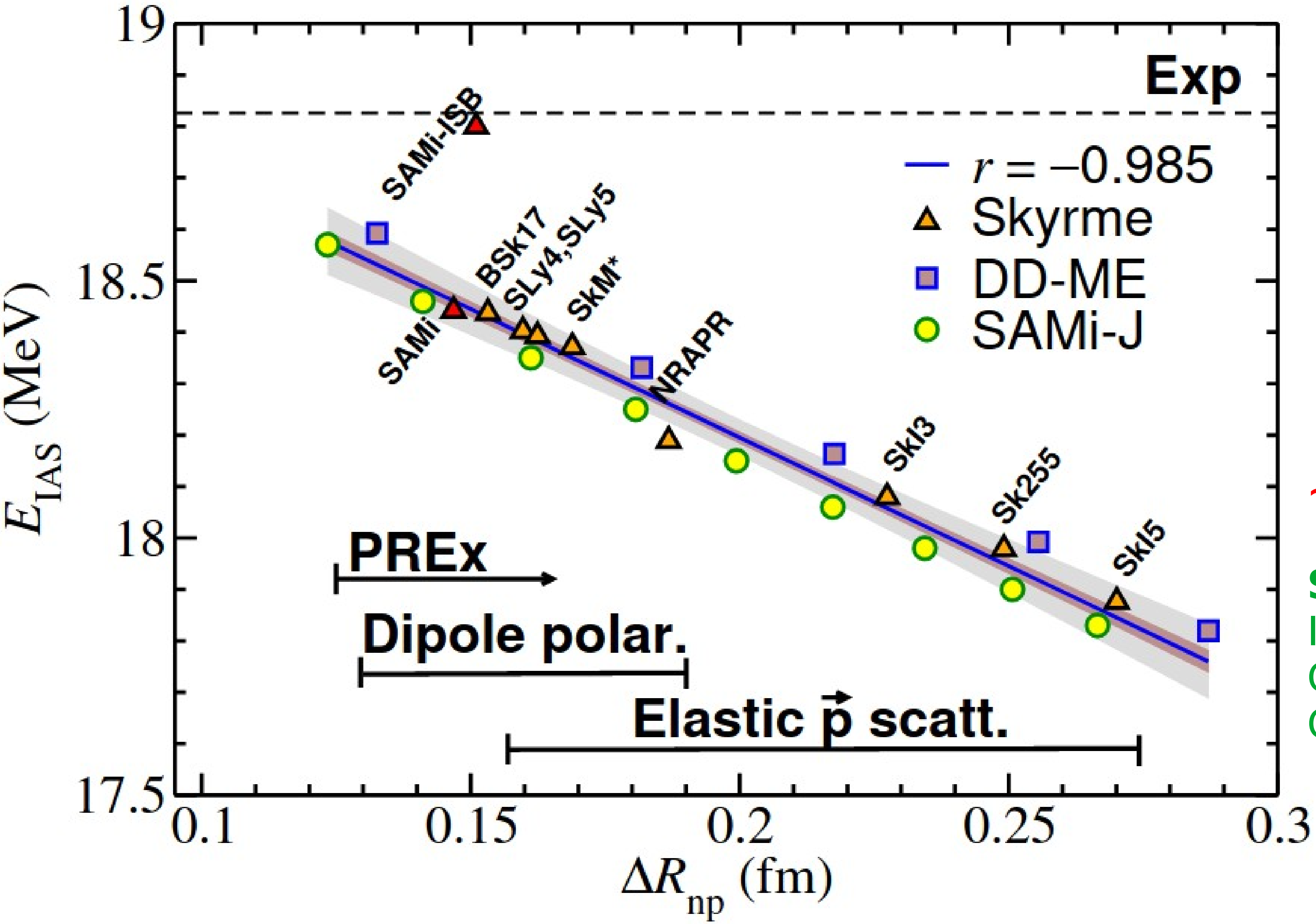
where $U_C^{direct}(\vec{r}) = \int \frac{e^2}{|\vec{r}_1 - \vec{r}|} \rho_{ch}(\vec{r}_1)\, d\vec{r}_1$

- Assuming also a uniform neutron and proton distributions of radius R_n and R_p respectively, and $\rho_{ch} \approx \rho_p$ one can find

$$E_{IAS} \approx E_{IAS}^{C,direct} \approx \frac{6}{5}\frac{Ze^2}{R_p}\left(1 - \sqrt{\frac{5}{12}\frac{N}{N-Z}\frac{\Delta r_{np}}{R_p}}\right)$$

One may expect: **the larger the Δr_{np} the smallest E_{IAS}**

From Earth: Isobaric Analog State and the breaking of isospin symmetry



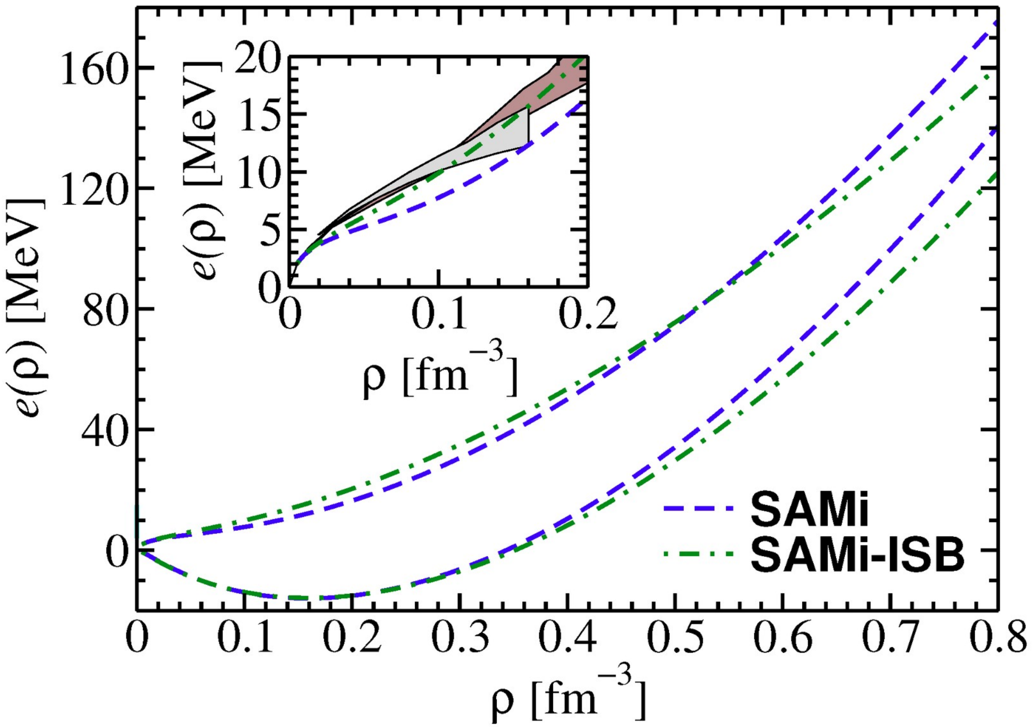
Exp errors in IAS ~ tens of keV

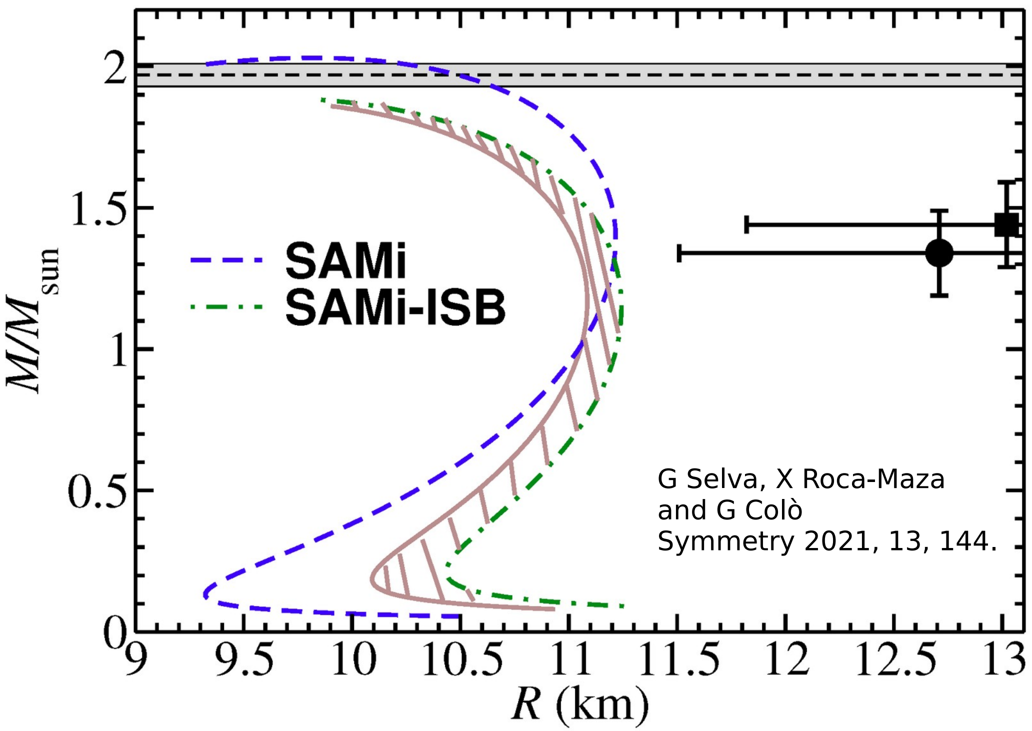
(or smaller)

Exp width IAS ~ hundreds of keV

SAMi-ISB: includes ISB effects due to Coulomb exchange, QED corrections and nuclear ISB

From Heaven: ISB effects on NS?

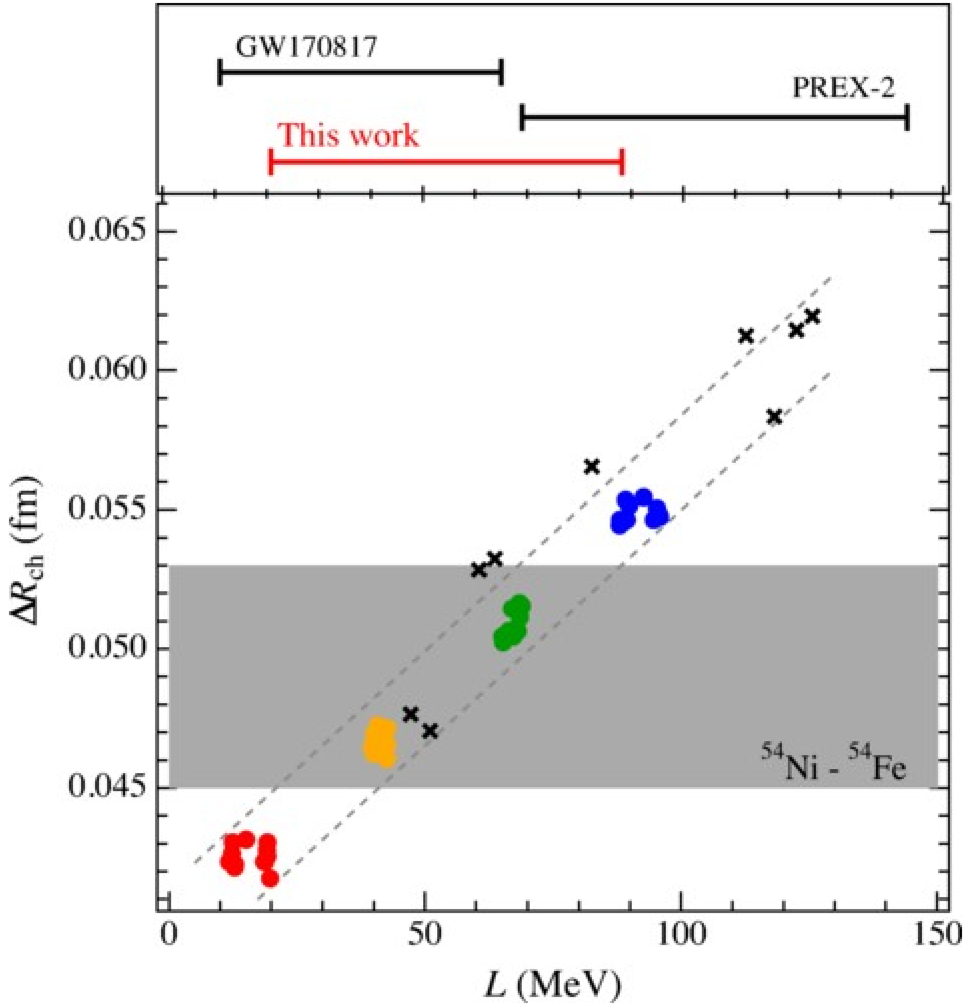




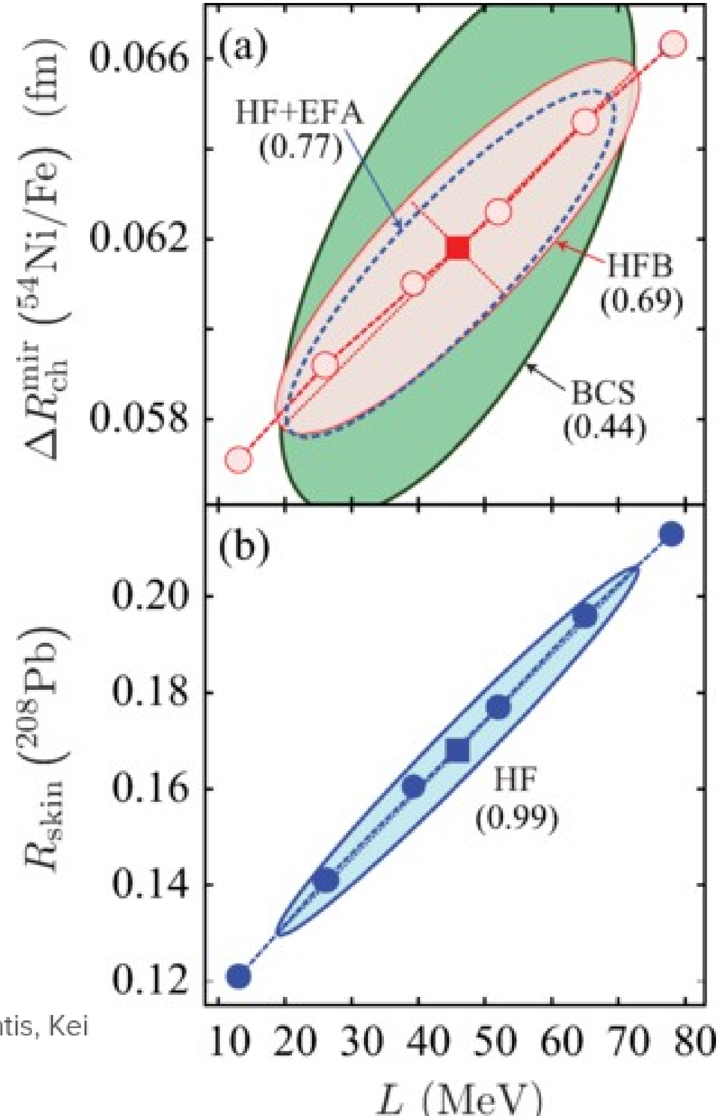
	M_{max}/M_{sun}	R_{max} [km]	$\rho^c_{1.4}$ [fm^{-3}]	$R_{1.4}$	$\Lambda_{1.4}$ [km]	$\xi_{1.4}$
SAMi	2.03	9.8	0.54	11.2	301	0.18
SAMi-ISB	1.88	9.8	0.59	11.2	261	0.19
SAMi-ISB ($u_0 = s_0 = 0$)	1.86	9.9	0.61	11.0	242	0.19

From Earth: charge radii difference in mirror mass nuclei

Isospin symmetry → $\Delta r_{ch} := r_{ch}(^{54}Ni) - r_{ch}(^{54}Fe) = \Delta r_{np}(^{54}Fe)$

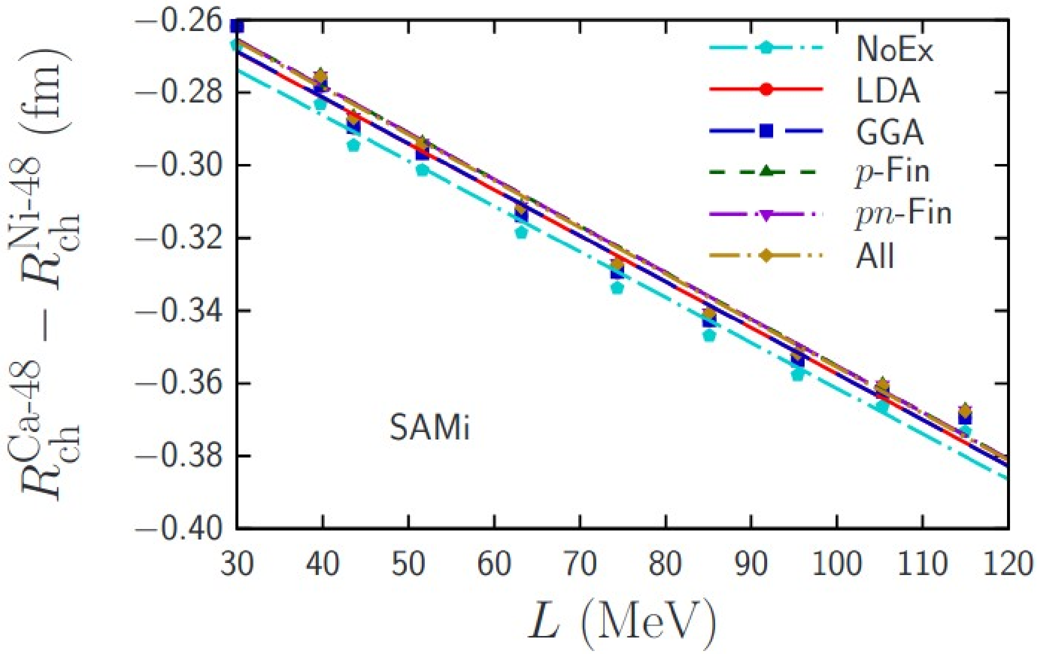


Skyy V. Pineda, Kristian König, Dominic M. Rossi, B. Alex Brown, Anthony Incorvati, Jeremy Lantis, Kei Minamisono, Wilfried Nörtershäuser, Jorge Piekarewicz, Robert Powel, and Felix Sommer
Phys. Rev. Lett. **127**, 182503 – Published 29 October 2021



Paul-Gerhard Reinhard and Witold Nazarewicz
Phys. Rev. C **105**, L021301 – Published 3 February 2022

From Earth: charge radii difference in mirror mass nuclei – ISB

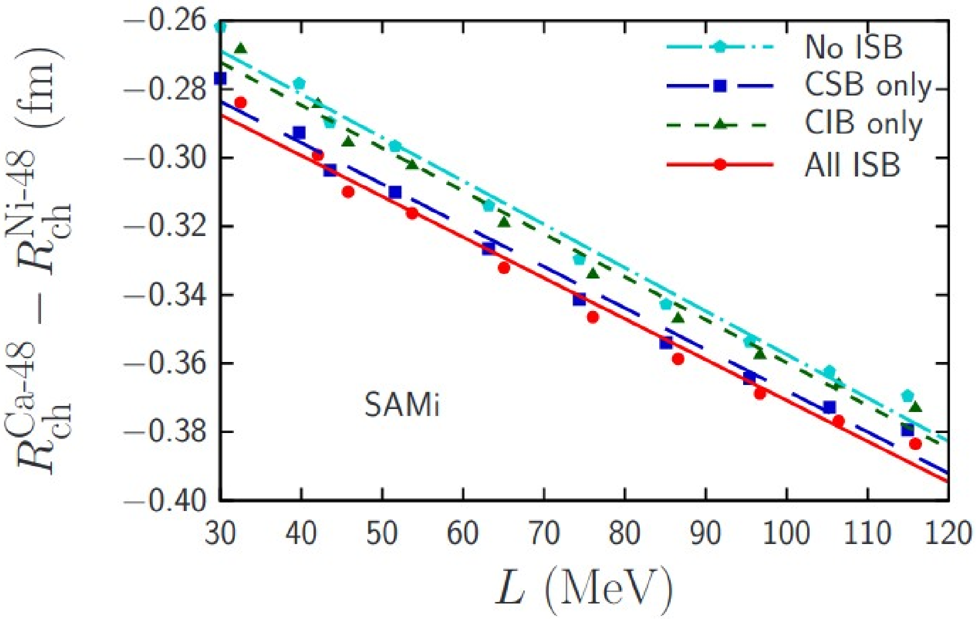


→ Accurate treatment of **Coulomb** (leading ISB in nuclei).

→ **No large effects found**.

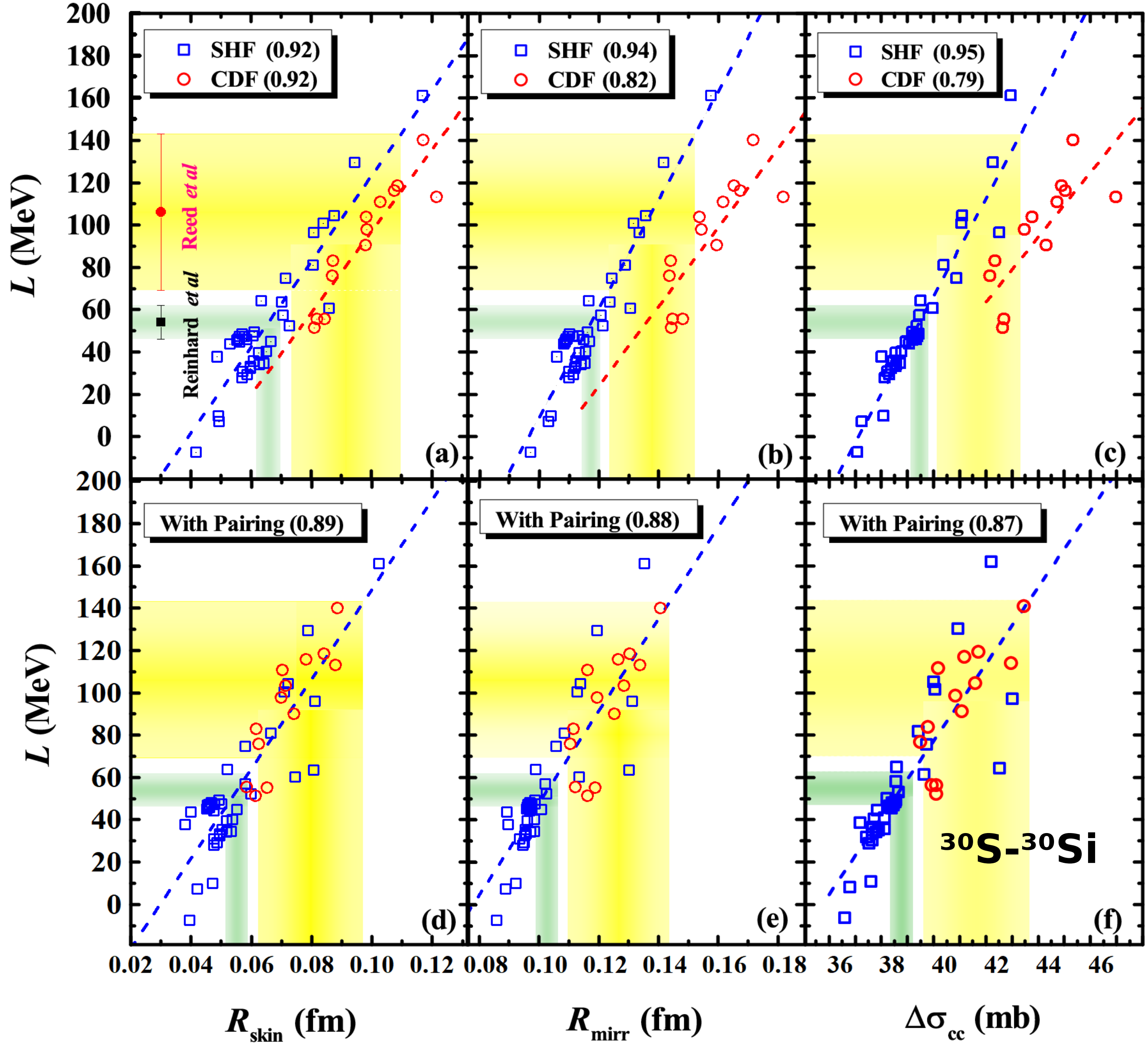
→ **Nuclear ISB** effect may impact on ***L*** determination by about **10 MeV** (SAMi-ISB)

→ Theoretical uncertainties must be estimated. Little knowledge about ISB in the medium



Tomoya Naito, Xavier Roca-Maza, Gianluca Colò, Haozhao Liang, Hiroyuki Sagawa, arXiv:2202.05035

From Earth: charge changing cross-sections



σ_{cc} → **probablity** to **remove protons** from incident nucleus **^{30}S** and **^{30}Si** in a target **^{1}H**

Glauber theory employed 900 MeV/nucleon

$\Delta\sigma_{cc}$ → leading term **Δr_{np}** (isospin symmetry assumed)

Jun-Yao Xua, Zheng-Zheng Li, Bao-Hua Sun, Yi-FeiNiu, XavierRoca-Maza, Hiroyuki Sagawa, Isao Tanihata, Phys. Lett. B on press. arXiv:2205.05276v2

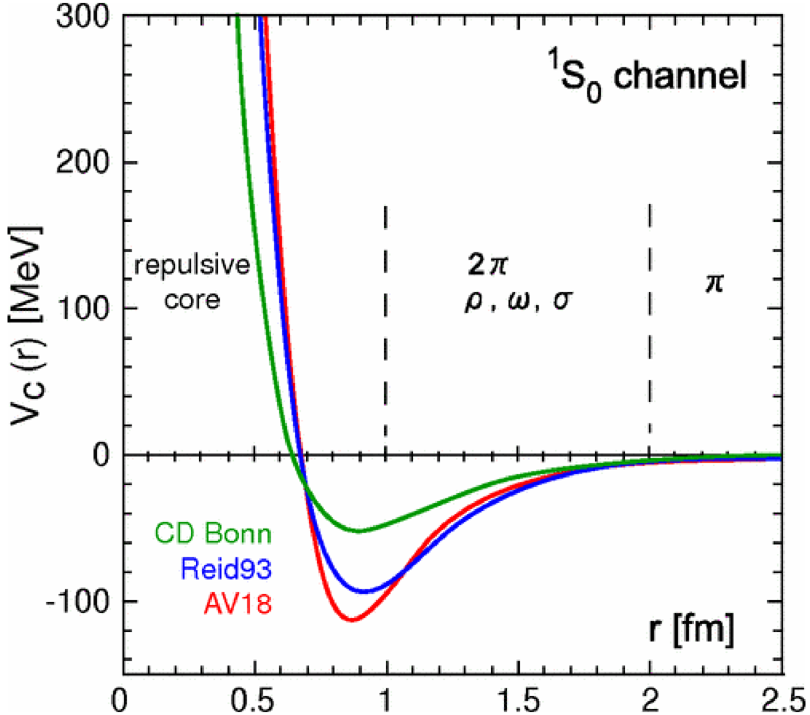
How are we dealing with the nuclear many-body problem?

(brief discussion on some of the Milano group efforts)

Nuclear Many-Body Problem

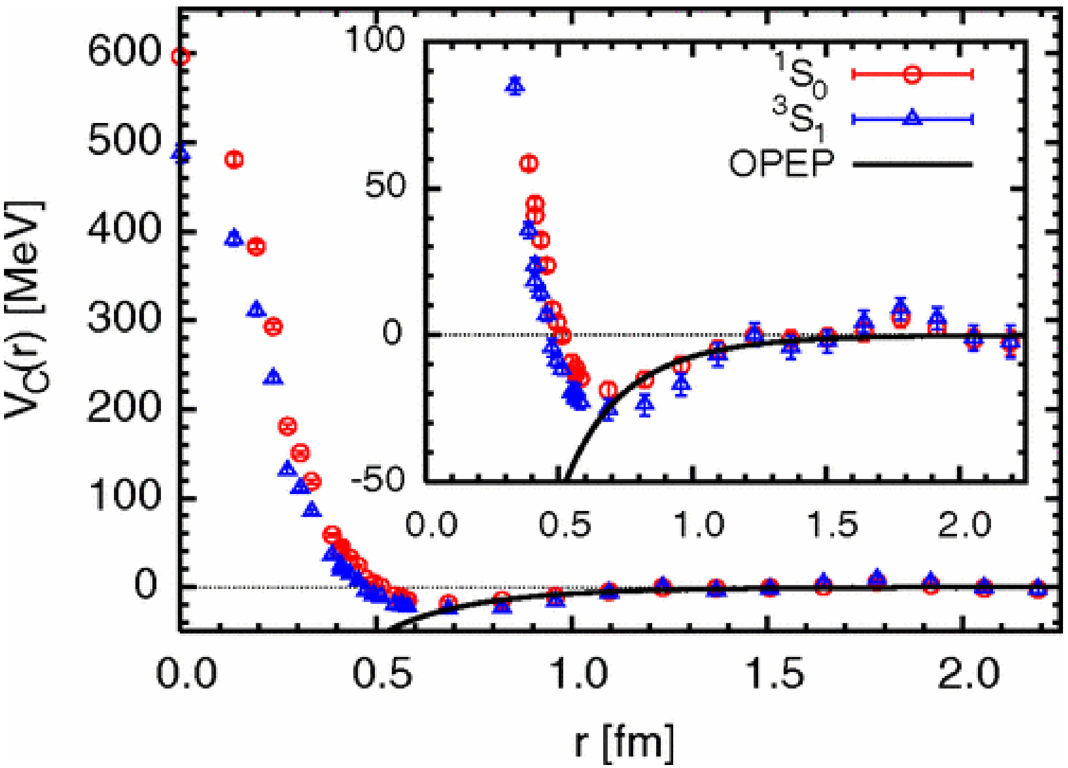
Underlying interaction: the “so called” **residual strong interaction** = **nuclear force** has **not** been **derived yet** (with the precision needed) from first principles as **QCD** is **non-perturbative** at the **low-energies** relevant for the description of nuclei.

Phenomenological



Nuclear Force from Lattice QCD - N. Ishii, S. Aoki, and T. Hatsuda
Phys. Rev. Lett. 99, 022001 (2007)

Lattice QCD (m_π/m_ρ~0.6)



Hohenberg-Kohn theorems
P.Hohenberg, W. Kohn, Phys. Rev. 136, B864 (1964)

→ Assuming a system of **interacting fermions** in a confining **external potential**, there exist a **universal** functional **F[ρ]** of the fermion density **ρ**:

$$E[\rho] = \langle \Psi | T + V + V_{\rm ext} | \Psi \rangle = F[\rho] + \int V_{\rm ext}(r)\rho(r) d\vec{r}$$

→ and it can be shown that

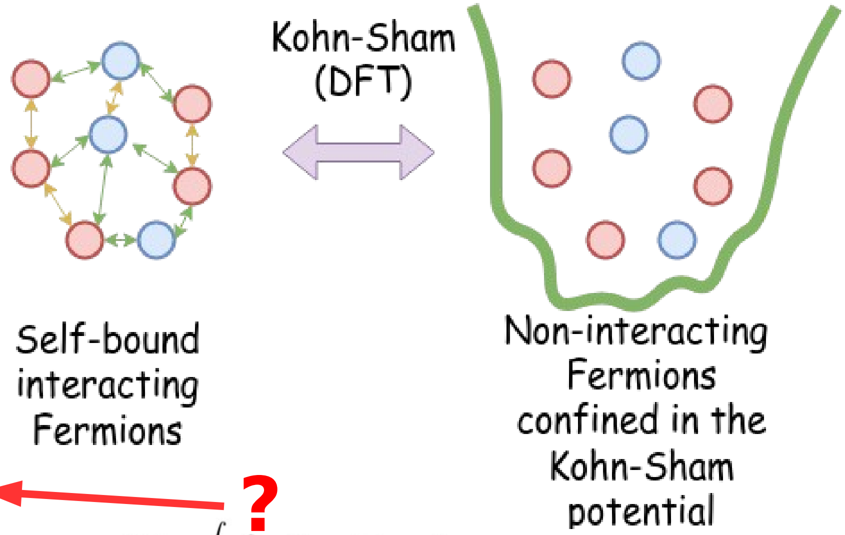
$$\min_{\Psi} \langle \Psi | T + V + V_{\rm ext} | \Psi \rangle = \min_{\rho} E[\rho]$$

so **E[ρ]** has a **minimum** for the **exact ground-state density** where it assumes the **exact energy** as a value.

Kohn-Sham realization
$F[\rho] \rightarrow T_{non\text{-}int.}[\rho]+V_{KS}[\rho]$

For **any interacting system**, there exists a **<u>local</u> single-particle potential V_{ks}(r)**, such that the **exact ground-state** density of the **interacting system** equals the **ground-state density** of the **auxiliary non-interacting system**:

$$\rho_{\text{exact}}(\vec{r}) = \rho_{\text{KS}}(\vec{r}) = \sum_{i=1}^{A} |\phi(\vec{r})|^2$$



where φ are single-particle orbitals and the total wave-function correspond to a Slater determinant. The **E[ρ] is unique**

$$E[\rho] = T[\rho] + \int V_{\text{KS}}(\vec{r})\rho(\vec{r})d\vec{r}$$

?

$$F[\rho] = \int d^3r \; f(\mathbf{r}, \rho, \nabla\rho, \ldots).$$

where **T[ρ]** is the **kinetic energy of the non-interacting system** and for which the variational equation

$$0 = \frac{\delta E[\rho]}{\delta\rho} = \frac{\delta T[\rho]}{\delta\rho} + V_{\text{KS}}$$

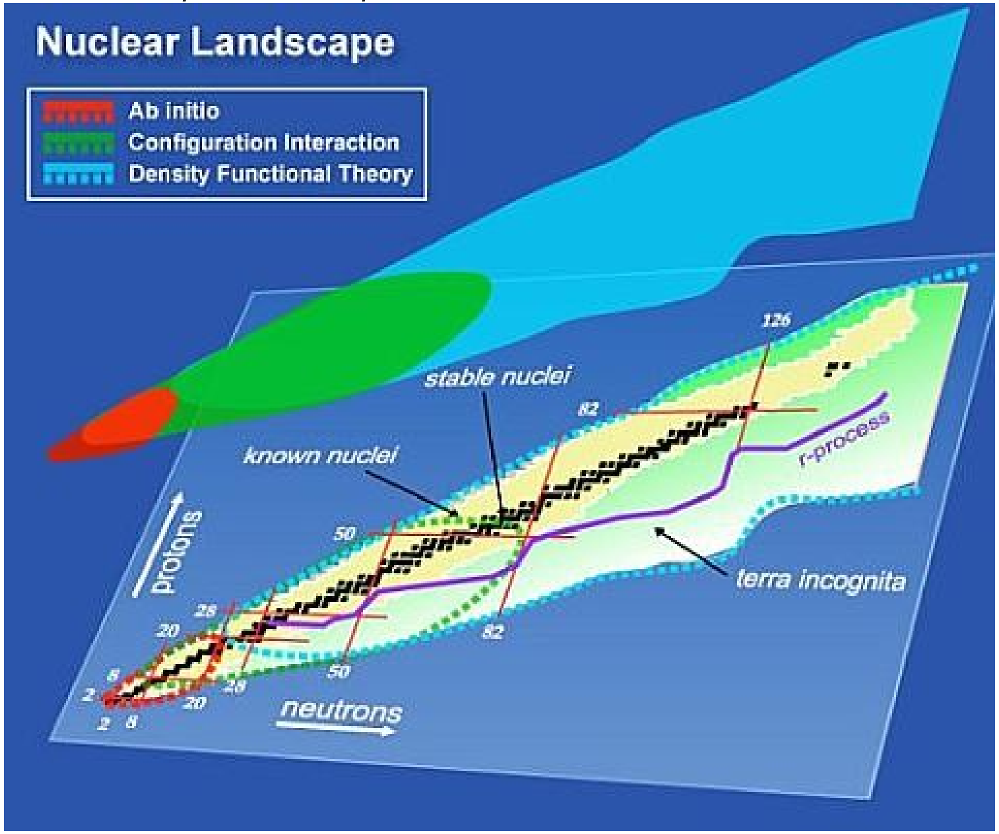
DFT Euler equation:

$$\frac{\delta T[\rho]}{\delta\rho(\mathbf{r})} + v_{KS}([\rho], \mathbf{r}) = \mu[\rho]$$

yields to the **exact ground state density and energy**

Advantadges and disadvantages of DFT

UNEDF http://unedf.mps.ohio-state.edu/



→ **ADVANTAGES OF DFT:**

- **exact theory** that can be applied to the **whole nuclear chart**
- **many-body** problem mapped onto a **one-body** problem without the need of explicitly involving inter-nucleon interactions!!! (computational cost and interpretation of observables in terms of single-particle properties)
- **HK generalised in (almost all) possible ways**: time dependence, degenerate ground-state, magnetic systems, finite T, relativistic case ...
- **any one body observable is within the DFT framework** (this includes also some sum rules related to nuclear excitations)

→ **DISADVANTAGES OF DFT:**

- various **proofs of HK theorems** do **not** give any clue on **how to build the functional**.
- **no** direct **connection** with **realistic NN or NNN interaction** if current approaches to EDF are not improved (some attempts already exist)
- **no systematic** way of **improvement** (evaluate syst. Errors) so far.

Avenues to improve EDFs? (@Milano)

→ We are working in two main directions:

1) **Inverse Kohn-Sham problem**: determine the V_{KS} and then E[ρ,...] from experimental and/or ab initio density distributions. With different **Bachelor and Master Thesis** students

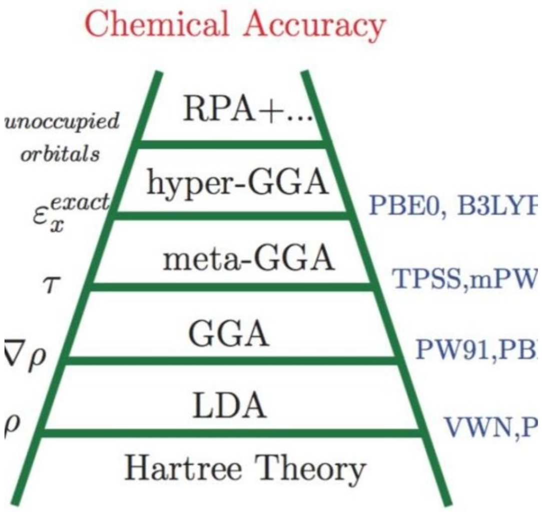
First step in the nuclear inverse Kohn-Sham problem: From densities to potentials

G. Accorto, P. Brandolini, F. Marino, A. Porro, A. Scalesi, G. Colò, X. Roca-Maza, and E. Vigezzi
Phys. Rev. C **101**, 024315 – Published 28 February 2020

Complete solution to the inverse Kohn-Sham problem: From the density to the energy

A. Liardi, F. Marino, G. Colò, X. Roca-Maza, and E. Vigezzi
Phys. Rev. C **105**, 034309 – Published 7 March 2022

2) **Mimic** strategy (**Jacob's Ladder**) in **many-electron systems** to systematically improve nuclear EDFs without using *phenomenological* parameters (as long as possible). With one **PhD** (Francesco Marino) and hopefully one postdoc in the future.



Nuclear energy density functionals grounded in *ab initio* calculations

F. Marino, C. Barbieri, A. Carbone, G. Colò, A. Lovato, F. Pederiva, X. Roca-Maza, and E. Vigezzi
Phys. Rev. C **104**, 024315 – Published 9 August 2021

Testing and understanding many-body theories

→ **Harmonic potential theorem:** should be fulfilled by any meaningful many-body theory. With **Bachelor** student and **PostDoc**.

Harmonic Potential Theorem: Extension to Spin-, Velocity-, and Density-Dependent Interactions

S. Zanoli, X. Roca-Maza, G. Colò, and Shihang Shen (申时行)

Phys. Rev. Lett. **123**, 112501 – Published 13 September 2019

→ **Exactly solvable models:** help in better understanding the involved approximations. With **Master** student and **PostDoc**.

LETTER

Extended Lipkin–Meshkov–Glick Hamiltonian

R Romano[1], X Roca-Maza[3,1,2], G Colò[1,2] and Shihang Shen(申时行)[1,2]

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Journal of Physics G: Nuclear and Particle Physics, Volume 48, Number 5

Citation R Romano *et al* 2021 *J. Phys. G: Nucl. Part. Phys.* **48** 05LT01

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→ Naito **Tomoya** (RIKEN),

→ Shihang **Shen** (Forschungszentrum Jülich)

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→ P.-G. **Reinhard** (University of Erlangen-Nürnberg)

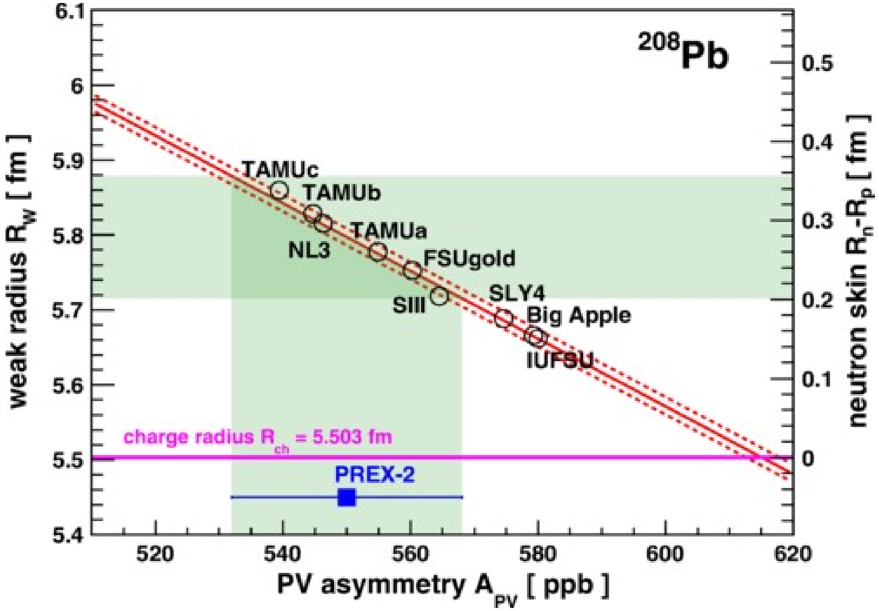
→ Yifei **Niu** (Lanzhou University)

→ Bao-Hua (Beihang University)

→ Haozhao **Liang** (University of Tokyo)

→ Witold **Nazarewicz** (FRIB and Michigan State University)

From Earth: PREx and CREx



D. Adhikari et al. Phys. Rev. Lett. 126, 172502 (2021)