Parity Violating Asymmetry and Dipole Polarizability in ^{208}Pb

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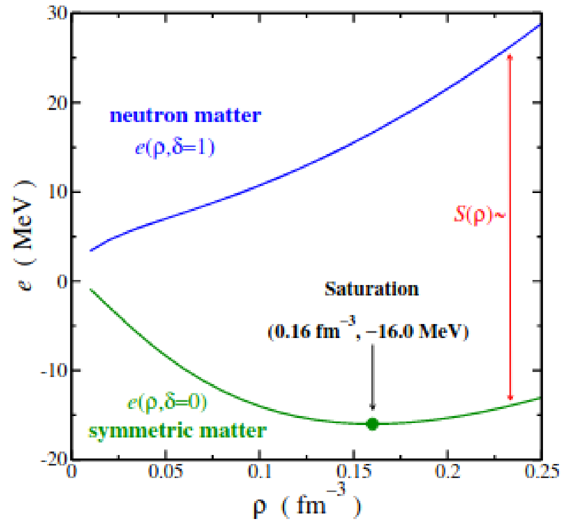
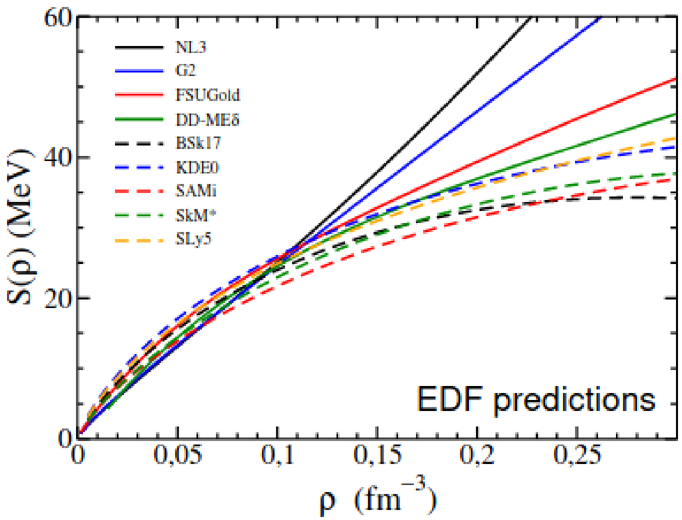
Motivation: nuclear EoS





$$e(\rho,\beta) = e(\rho,\beta=0) + S(\rho)\beta^2 + \mathcal{O}(\beta^4) \quad \text{where} \quad \beta \equiv \frac{\rho_n - \rho_p}{\rho}$$

$$J \equiv S(\rho_0); \quad L = 3\rho_0 \frac{\partial S(\rho)}{\partial \rho}\bigg|_{\rho=\rho_0}$$

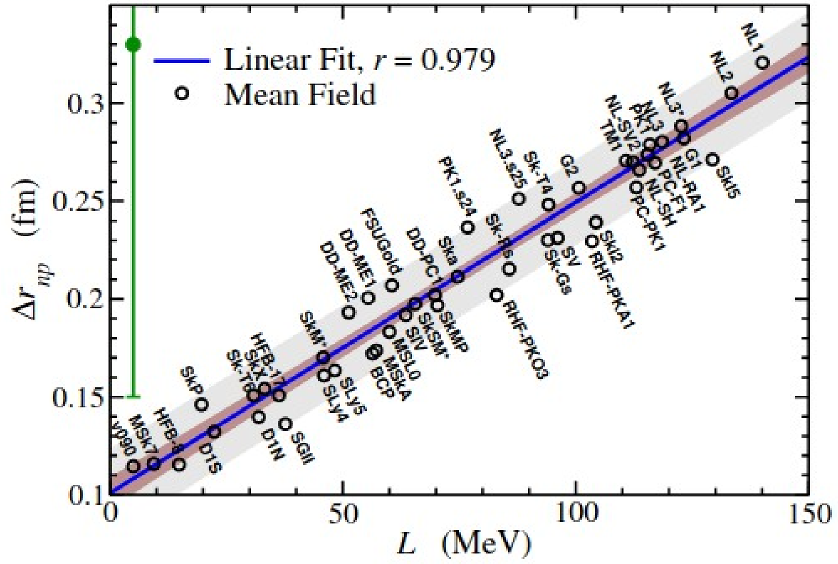
-Isovector properties not well determined in current EDFs

EDFs typically fitted to masses and charge radii

Motivation: neutron skin thickness

But how we can better constraint the isovector channel from observables? (Example)

Neutron skin thickness → is one of the most paradigmatic example of an **isovector sensitive observable**.



Physical Review Letters **106**, 252501 (2011)

$$\Delta r_{np} \equiv \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2} \sim \frac{1}{12}\frac{N-Z}{A}\frac{R}{J} L \; ; \qquad J \equiv S(\rho_0) \text{ and } L \equiv 3\rho_0 p_0^{neut}$$

But the **neutron skin thickness** is **not** directly **observable**

The neutron skin can be proben by the Parity Violating Asymmetry

- → Electrons interact by exchanging a γ **(couples to p)** or a Z_0 **boson (couples to n)**
- → Ultra-relativistic electrons, **depending on their helicity ($\pm$)**, will interact with the nucleus **seeing a slightly different potential**: **Coulomb $\pm$ Weak**
- → $$\boxed{A_{pv} \equiv \frac{d\sigma_+/d\Omega - d\sigma_-/d\Omega}{d\sigma_+/d\Omega + d\sigma_-/d\Omega}} \sim \frac{\text{Weak}}{\text{Coulomb}}$$
- → Input for the calculation are the ρ_p and ρ_n **(main uncertainty)** and nucleon form factors for the e-m and the weak neutral current.

Electron motion governed by the Dirac equation:

$$[\vec{\alpha}\cdot\vec{p} + \beta m_e + V(r)]\psi = E\psi$$

where $V(r) = V_C(r) + \gamma^5 V_W(r)$

Dirac equation for helicity states ($m_e \approx 0$)

$$[\vec{\alpha}\cdot\vec{p} + (V_C(r) \pm V_W(r))]\psi_\pm = E\psi_\pm$$

Parity Violating Asymmetry:
definition and simple model (PWBA; $F_W \approx F_n$; and $F_{ch} \approx F_p$)

Within the Plane Wave Born Approximation,

$$A_{pv} = \frac{G_F q^2}{4\pi\alpha\sqrt{2}}\left[4\sin^2\theta_W + \frac{F_n(q) - F_p(q)}{F_p(q)}\right]$$

... which depends on $F_n(q) - F_p(q)$. For $q \to 0$, it is approximately,

$$\begin{aligned} -\frac{q^2}{6}\left(\langle r_n^2\rangle - \langle r_p^2\rangle\right) &= -\frac{q^2}{6}\left[\Delta r_{np}\left(\langle r_n^2\rangle^{1/2} + \langle r_p^2\rangle^{1/2}\right)\right] \\ &= -\frac{q^2}{6}\left(2\langle r_p^2\rangle^{1/2}\Delta r_{np} + \Delta r_{np}^2\right) \end{aligned}$$

variation of A_{pv} at a fixed q dominated by the variation of Δr_{np}. $F_p(q)$ well fixed by experiment

Dipole Polarizability is also a sensitive isovector indicator: definition

The **linear response** or dynamic polarizability of a **nuclear system excited** from its g.s., $|0\rangle$, to an excited state, $|\nu\rangle$, due to the **action of an external isovector oscillating field** (dipolar in our case) of the form $(Fe^{iwt} + F^{\dagger}e^{-iwt})$:

$$F_{JM} = \sum_{i}^{A} r^J Y_{JM}(\hat{r})\tau_z(i) \ (\Delta L = 1 \to \text{Dipole})$$

is proportional to the **static polarizability** for small oscillations

$\alpha = (8\pi/9)e^2 m_{-1} = (8\pi/9)e^2 \sum_{\nu} |\langle \nu|F|0\rangle|^2/E$ where m_{-1} **is the inverse energy weighted moment** of the **strength function**, defined as, $S(E) = \sum_{\nu} |\langle \nu|F|0\rangle|^2 \delta(E - E_{\nu})$

Dipole Polarizability: simple model

electric polarizability measures tendency of the nuclear charge distribution to be distorted ($\alpha \sim \frac{\text{electric dipole moment}}{\text{external electric field}}$)

- The dielectric theorem establishes that the m_{-1} moment can be computed from the expectation value of the Hamiltonian in the constrained ground state $\mathcal{H}' = \mathcal{H} + \lambda\mathcal{D}$.

Adopting the Droplet Model:

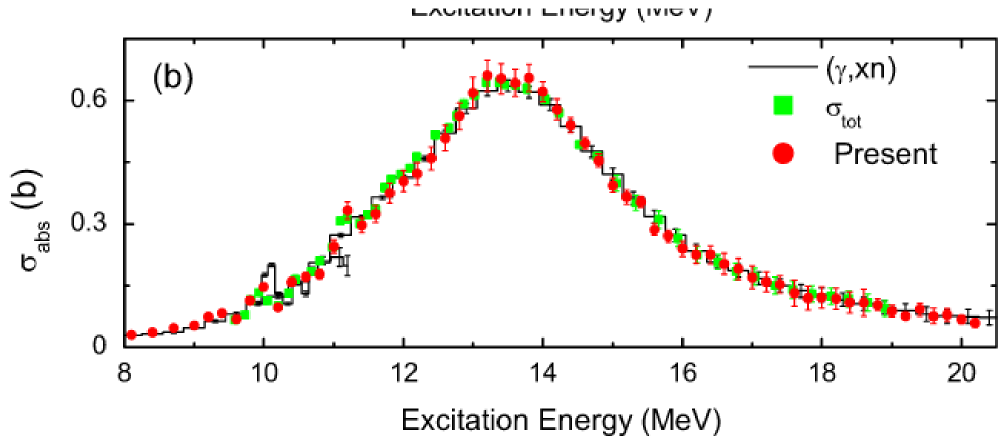
$$m_{-1} \approx \frac{A\langle r^2\rangle^{1/2}}{48J}\left(1 + \frac{15}{4}\frac{J}{Q}A^{-1/3}\right)$$

within the same model, connection with the neutron skin thickness:

$$\alpha_D \approx \frac{A\langle r^2\rangle}{12J}\left[1 + \frac{5}{2}\frac{\Delta r_{np} + \sqrt{\frac{3}{5}\frac{e^2 Z}{70J}} - \Delta r_{np}^{\text{surface}}}{\langle r^2\rangle^{1/2}(I - I_C)}\right]$$

Recent exp. results on A_{pv} and α_D in ^{208}Pb

Tamii et al. PRL107 062502 (2011)



$$\alpha_D = \frac{\hbar c}{2\pi^2 e^2} \int \frac{\sigma_{\mathrm{abs}}}{\omega^2} d\omega,$$

$$\boxed{\alpha_D(^{208}\mathrm{Pb}) = 19.6 \pm 0.6\ \mathrm{fm}^3}$$

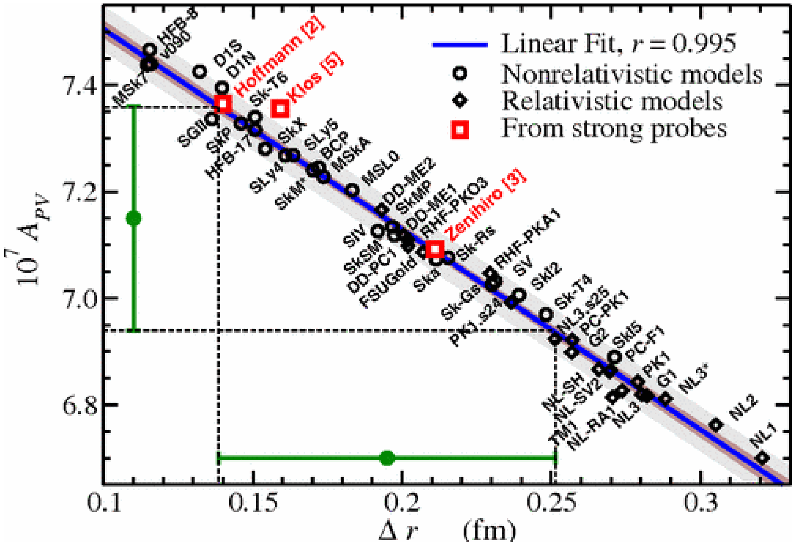
PREx: D. Adhikari et al. (PREX Collaboration) Phys. Rev. Lett. 126, 172502 (2021)

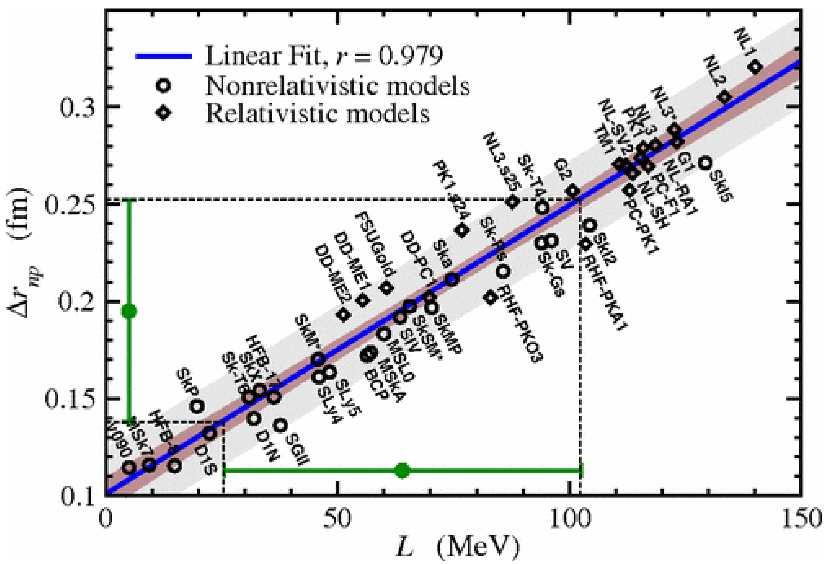
Our final results for $A_{\mathrm{PV}}^{\mathrm{meas}}$ and F_W with the acceptance described by $\epsilon(\theta)$ and $\langle Q^2 \rangle = 0.00616\ \mathrm{GeV}^2$ are

$$\boxed{A_{\mathrm{PV}}^{\mathrm{meas}} = 550 \pm 16\,(\mathrm{stat}) \pm 8\,(\mathrm{syst})\ \mathrm{ppb}}$$

$$F_W(\langle Q^2 \rangle) = 0.368 \pm 0.013\,(\mathrm{exp}) \pm 0.001\,(\mathrm{theo}),$$

Analysis based on EDFs correlations between A_{pv}, ΔR_{ch} and L

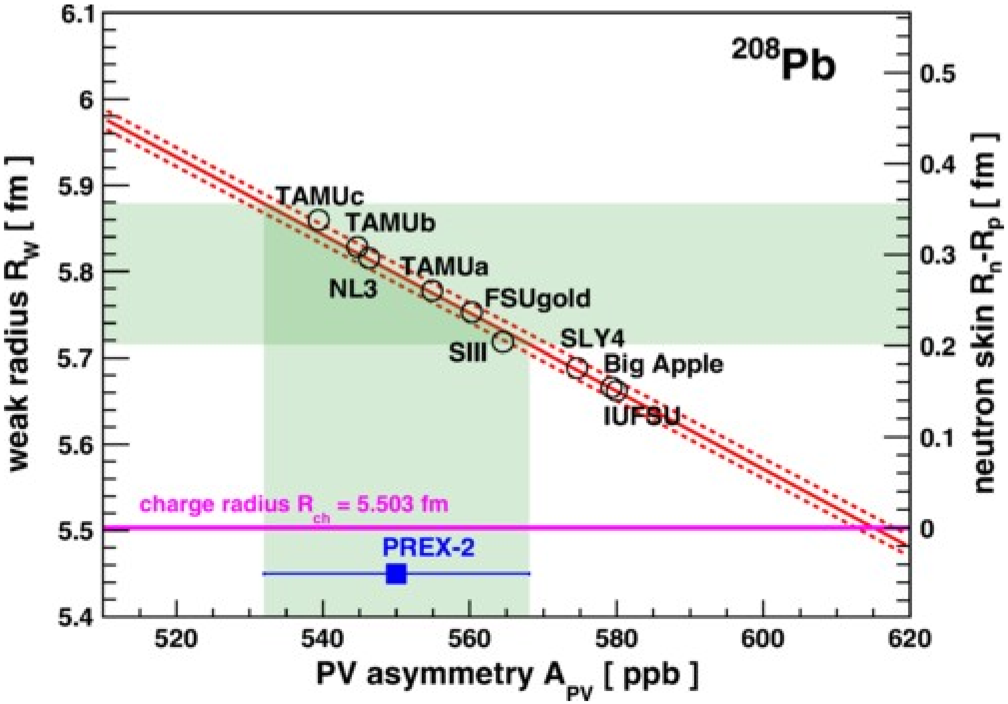




X. Roca-Maza, et al. Phys. Rev. Lett. 106, 252501 (2011)

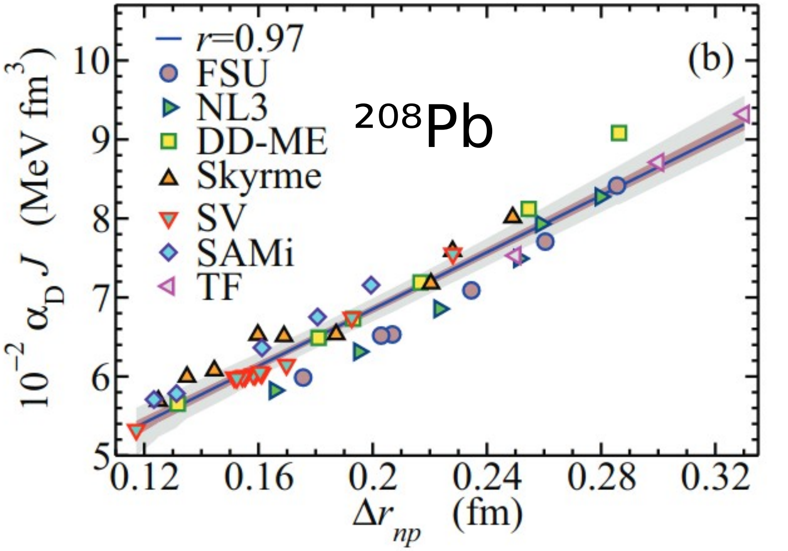
EDFs callibrated to **reproduce** the **binding energy** and **charge radii** of some **selected nuclei**. (In some cases pseudo-observables related to the EoS are also used)

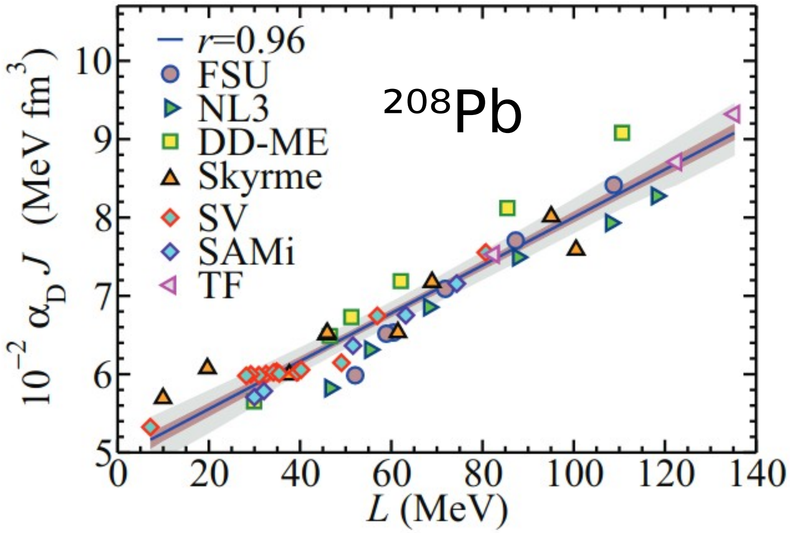
$$\boxed{R_n - R_p = 0.283 \pm 0.071 \text{ fm.}}$$



D. Adhikari et al. Phys. Rev. Lett. 126, 172502 (2021)

Analysis based on EDFs correlations between α_D, ΔR_{ch} and L





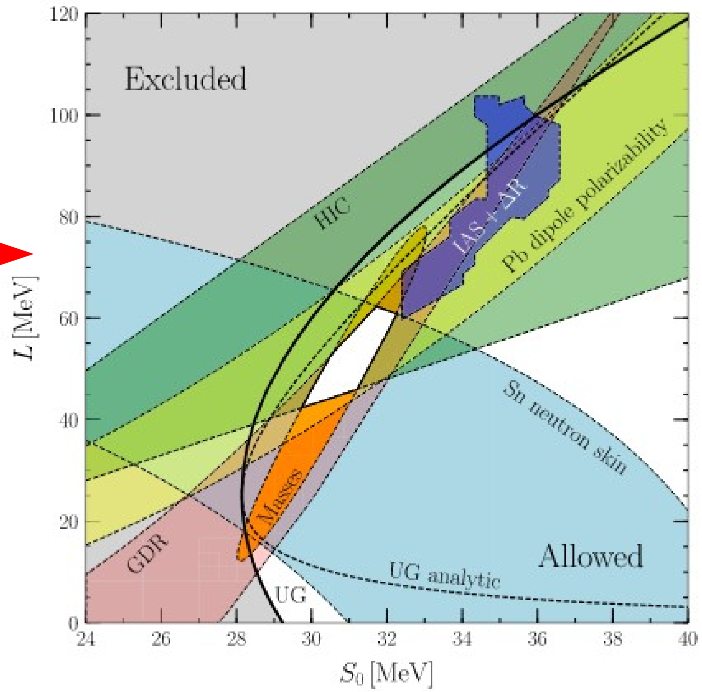
X. Roca-Maza,et al. Phys. Rev. C 88, 024316 (2013)

$$\boxed{\Delta r_{np} = 0.165 \pm (0.009)_{\mathrm{expt}} \pm (0.013)_{\mathrm{theor}} \pm (0.021)_{\mathrm{est}}\ \mathrm{fm}.}$$

$$J = (24.5 \pm 0.8) + (0.168 \pm 0.007)L.$$

X. Roca-Maza,et al. Phys. Rev. C 92, 064304 (2015)

Typical plot containing different constraints from different analysis performed in different ways on two non-observable quantities



Ingo Tews et al 2017 ApJ 848 105

New analysis taking into account model error quantification

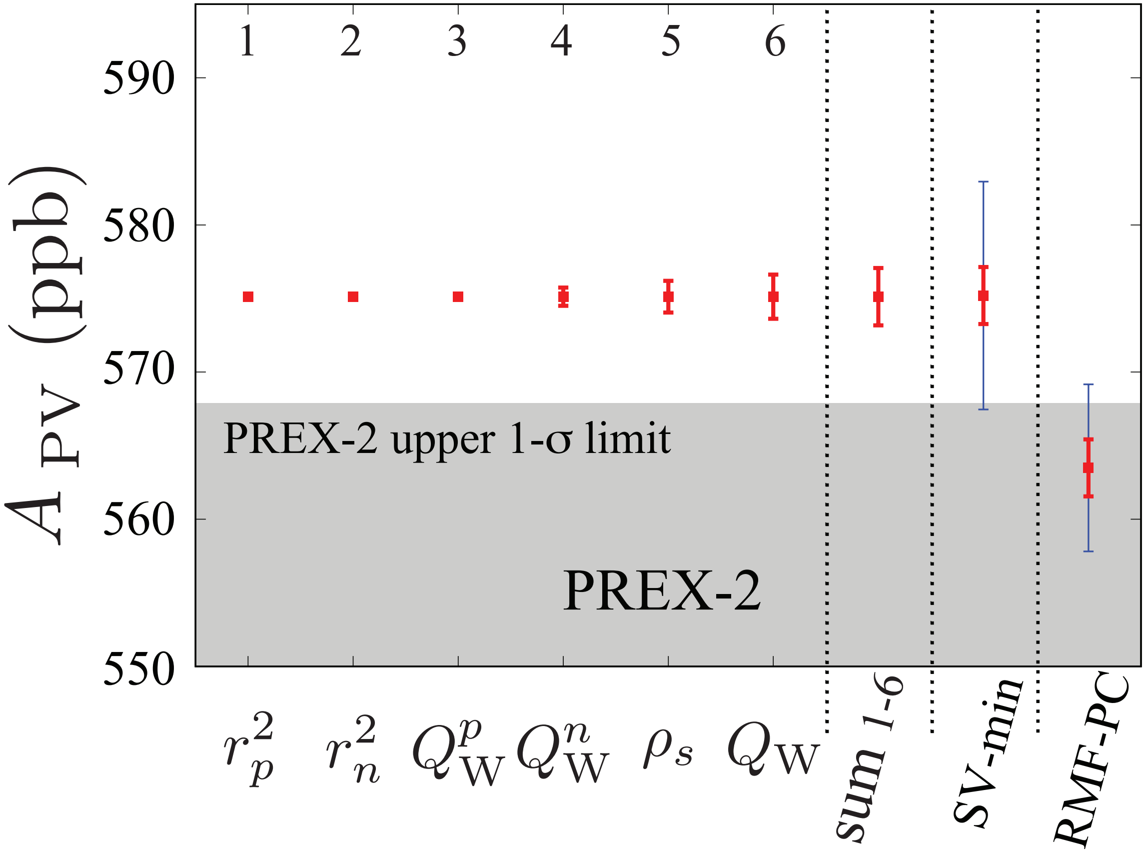
How to improve current analysis?

- Include theoretic **statistical errors** and **correlations** within a given **EDF** parametrization
- **Fitting** procedure including experimental data not only on **B** and $\mathbf{R}_{ch}$ but also on $\mathbf{A}_{pv}$ and/or $\boldsymbol{\alpha}_D$ (**“informed” EDFs**)

  …
- **Generalizing** available **EDFs?**
- Other issues related to theory**?**

PREx: theo. uncertainty budget for A_{PV}

Uncertainties in the determination of the Form Factors is smaller than typical EDFs statistical uncertainties



Parameter	Value
$\langle r_p^2 \rangle$ (fm^2)	0.726 ± 0.019
$\langle r_n^2 \rangle$ (fm^2)	-0.1161 ± 0.0022
μ_p	$2.792\,847$
μ_n	-1.9130
$Q_p^{(W)}$	0.0713 ± 0.0001
$Q_n^{(W)}$	-0.9888 ± 0.0011
ρ_s	-0.24 ± 0.70
κ_s	-0.017 ± 0.004
$Q_{126,82}^{(W)}$	-117.9 ± 0.3

Thin blue bars: statistical model uncertainties (related to neutron and proton densities)

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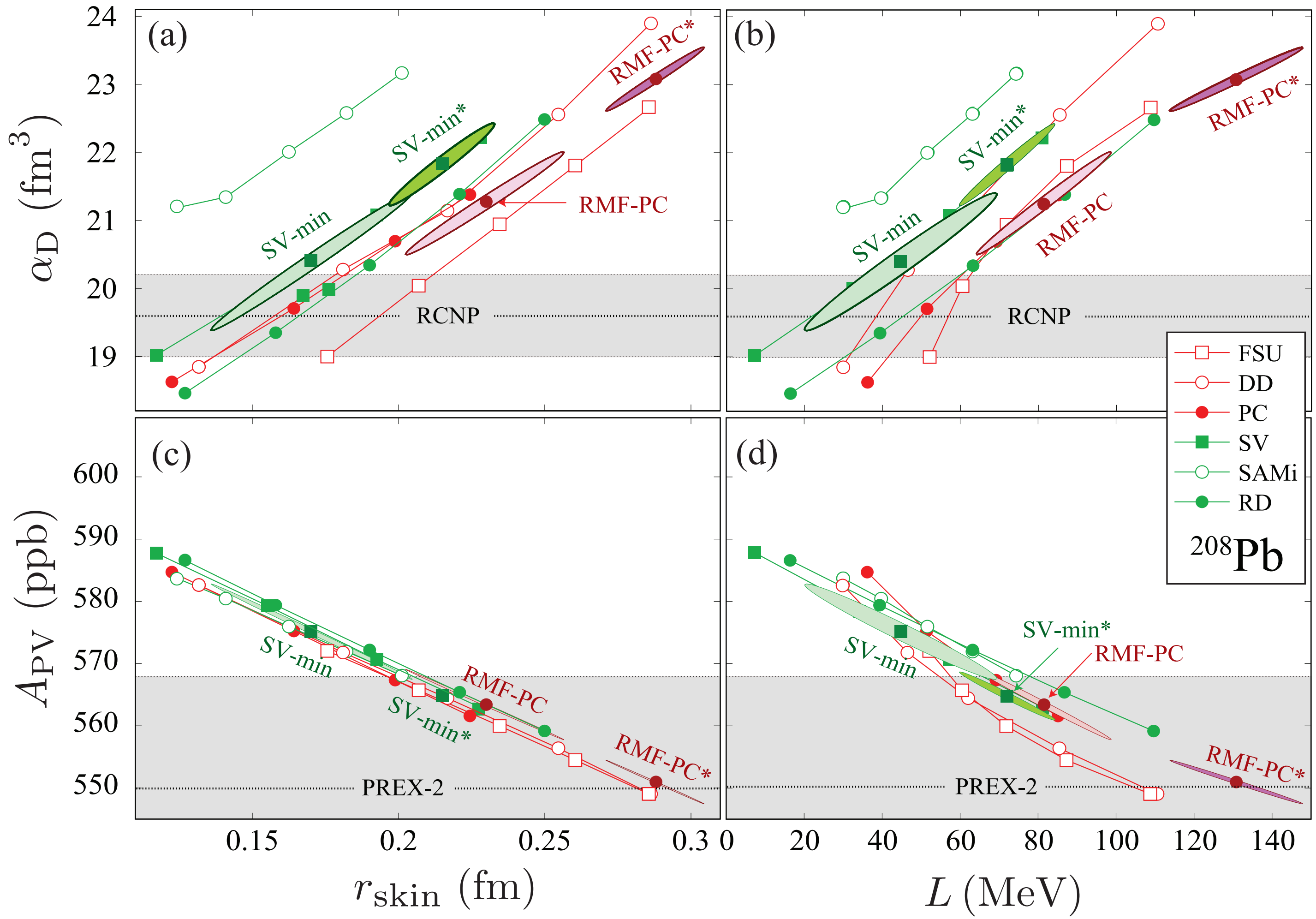
EDF predictions with error ellipsoids for A_{pv} and α_D in ^{208}Pb

Correlation ellipsoids within each **EDF** show **similar correlations** than the **systematic** study with many **EDFs ↔ have we learnt something?**

FSU→ Non linear Walecka model (Rel)
DD→ Meson-exchange with density dependent couplings (Rel)
PC→ zero-range density dependent couplings (Rel)
SV&SAMi→ Skyrme (zero-range, Non-rel)
RD→ Skyrme with modified density dependence (ρ^integer powers)

Models with a "*" contain A_{pv} in the fitting protocol



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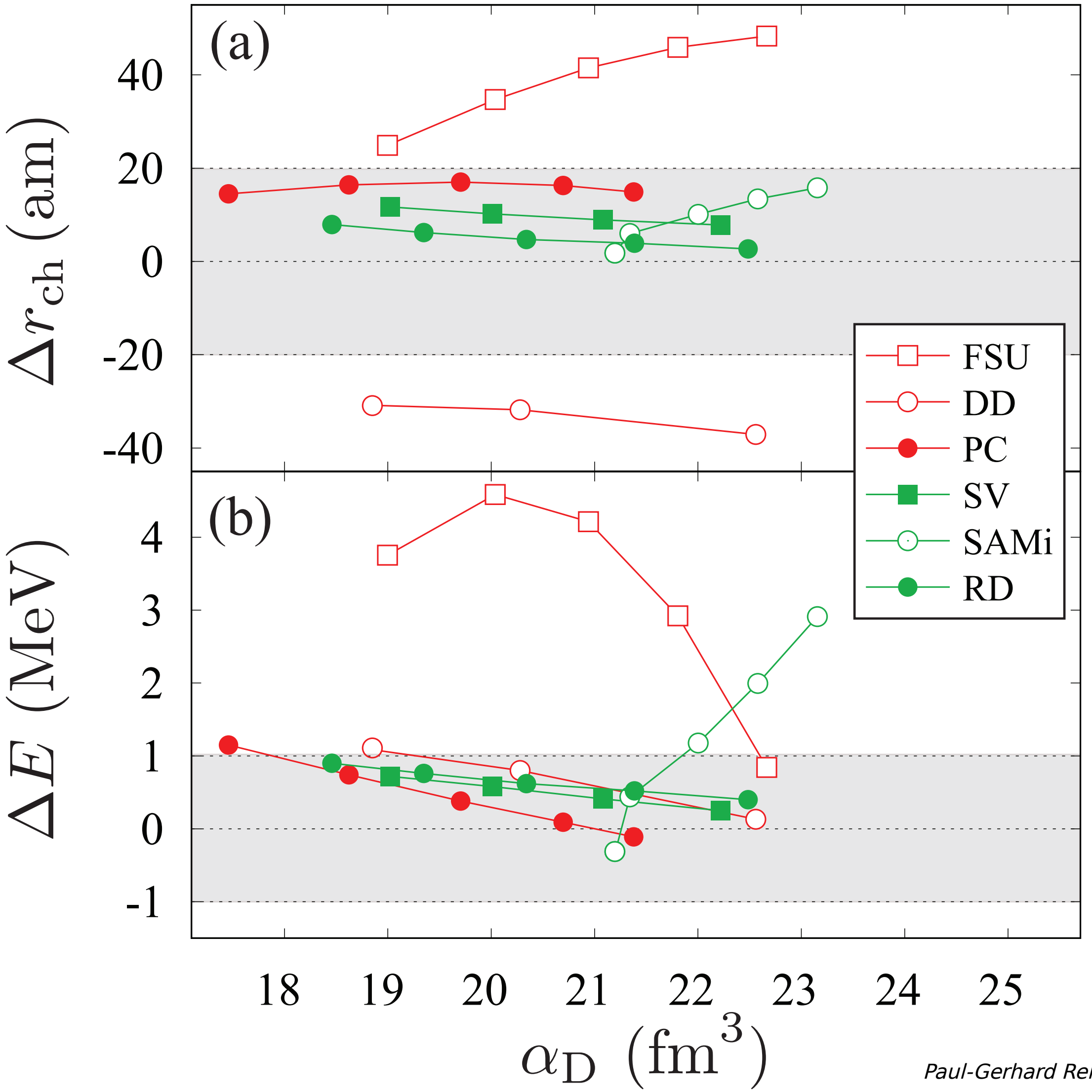
How these models perform for B and R_{ch} in ^{208}Pb?



→ The **residuals** of the charge **radius** (a) and binding **energy** (b) of ^{208}**Pb** for the theoretical models.

→ The **grey bands** around the perfect match indicate the typical performance of EDFs (i.e. **typical** r.m.s. **deviation** taken over all nuclei where correlation effects are small)

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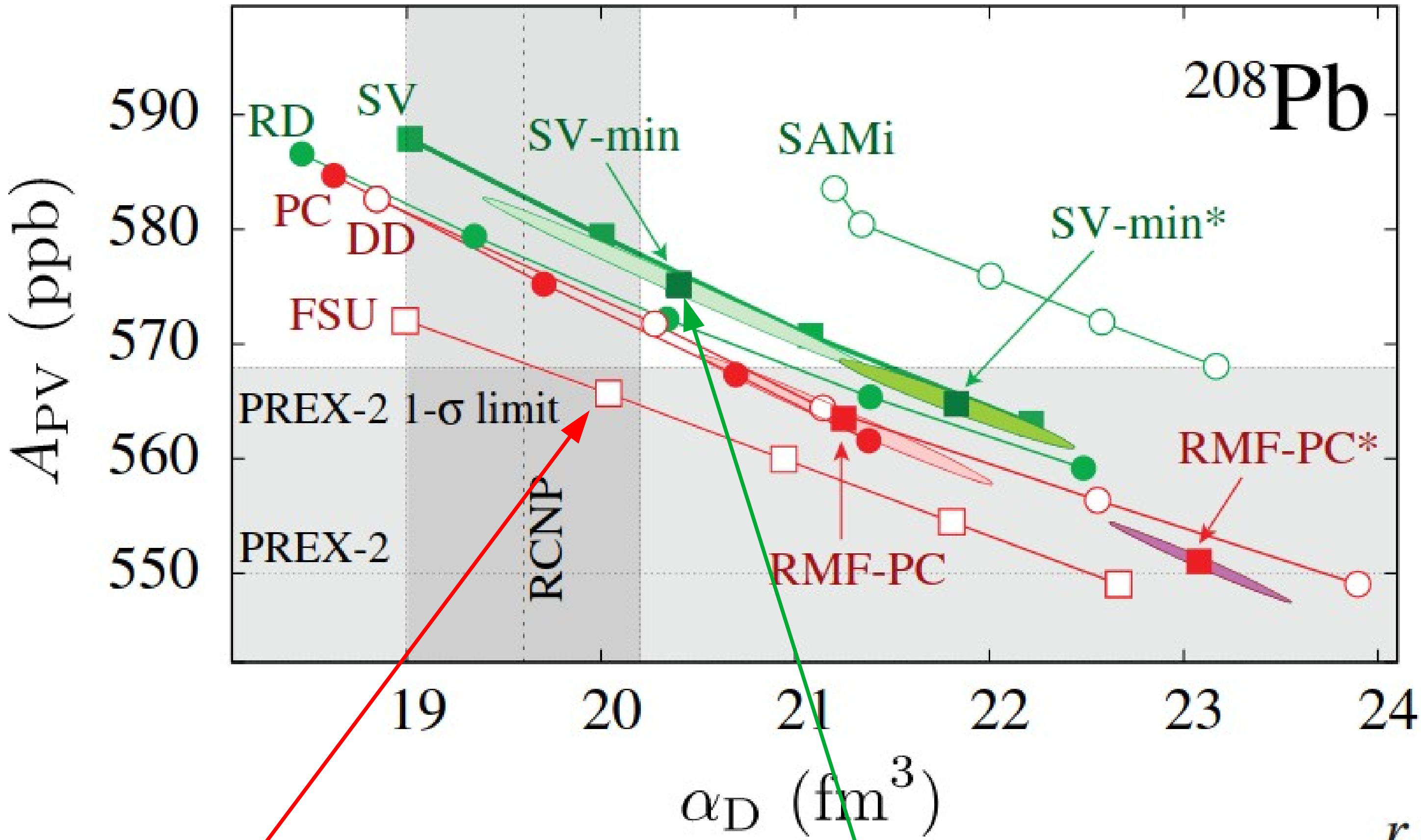
A_{pv} versus α_D in well callibarted EDFs



SV-min, suitable model to predict EoS around ρ_0 and Δr_{np}

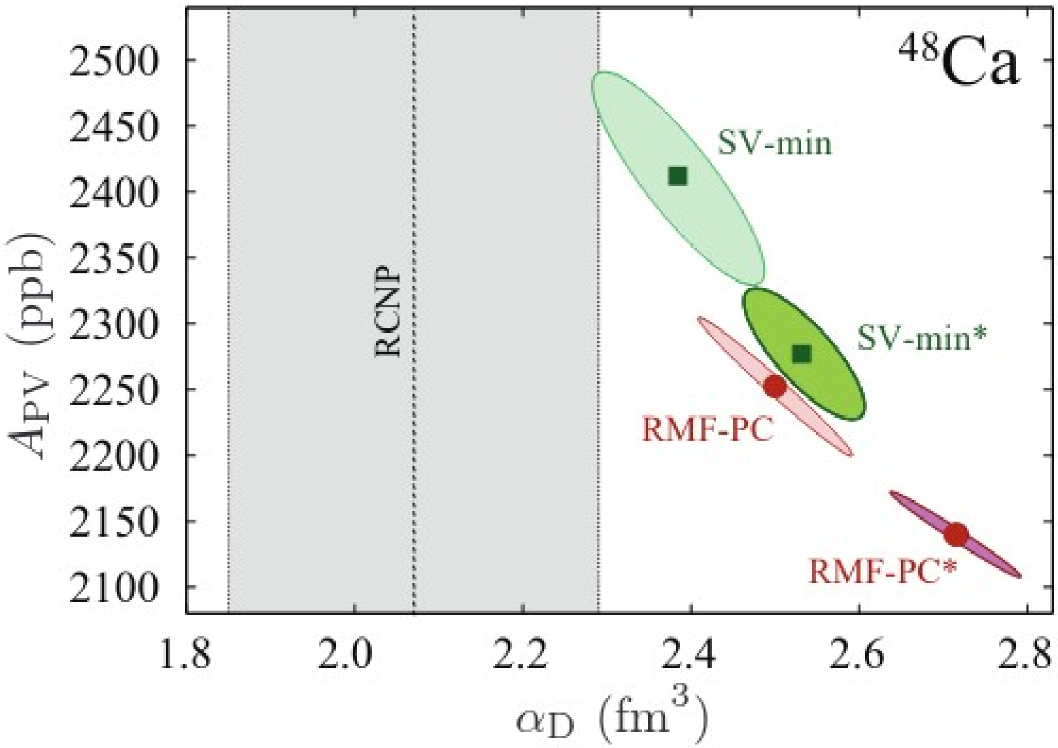
$$r_{skin} = 0.19 \pm 0.02 \text{ fm}$$

$$L = 54 \pm 8 \text{ MeV}$$

B and **Rch** in 208Pb not as good as other EDFs

Theoretical and experimental 1σ errors overlap for SV-min

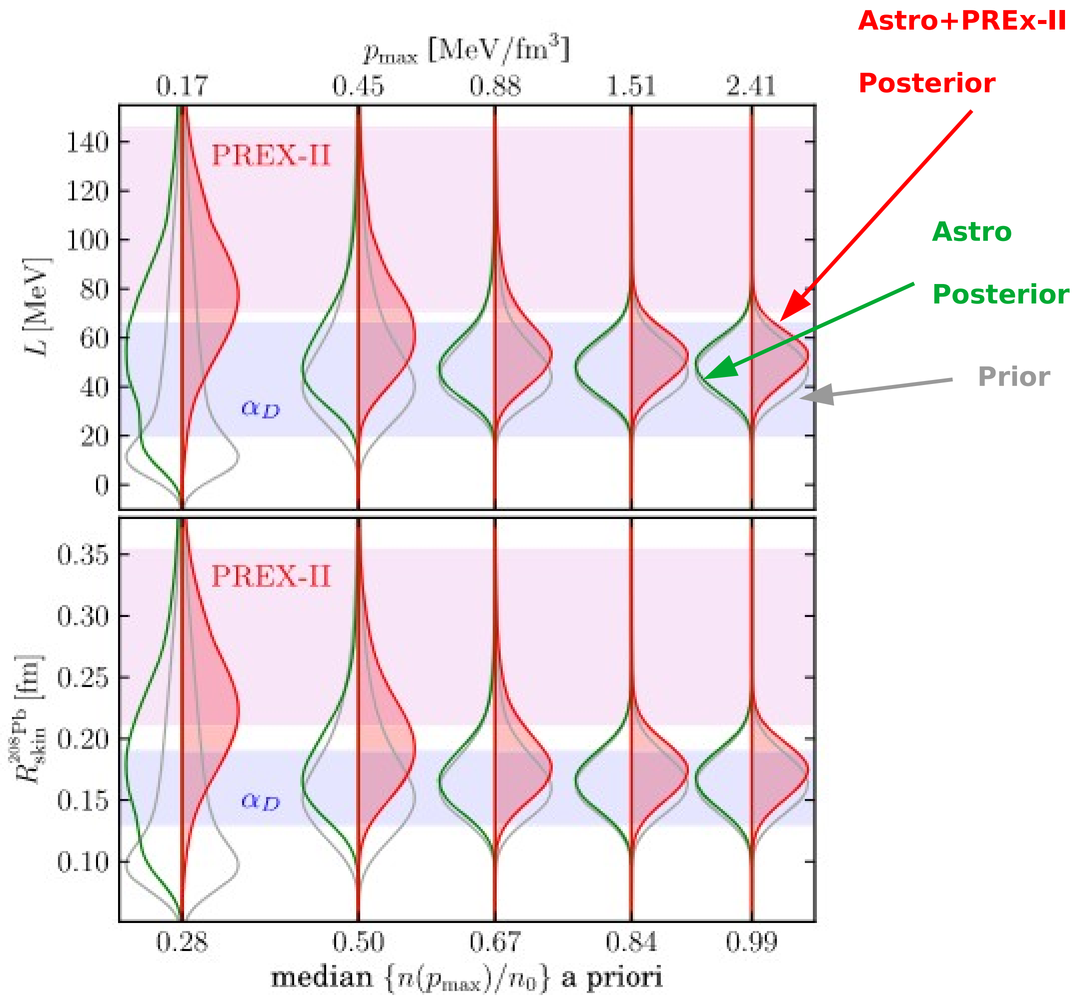
Predictions for CREx $A_{\mathrm{PV}}(^{48}\mathrm{Ca}) = 2400 \pm 60\,\mathrm{ppb}.$



Preliminary result form CREx A_{pv} =2568.6±113.2 ppb

Our prediction is 1.5σ away from experiment

Other example: minimal model assumptions and statistical analysis



Reed Essick, et al. Phys. Rev. Lett. 127, 192701 (2021)

Non-parametric **equation of state** representation derived from **observations of neutron stars** with minimal modeling assumptions.

The resulting **astrophysical constraints** from heavy pulsar masses, LIGO/Virgo, and NICER clearly **favor "small"** values of the **neutron skin** and **L**.

Combining astrophysical data with **PREX-II** and **chiral effective field theory** constraints yields

J=33.0 ± 2.0 MeV

L=53 ± 15MeV

R_{skin}=0.17 ± 0.04 fm

Conclusions

- **Current EDFs** show strong ***systematic*** and ***statistic*** **correlations** between $\mathbf{A_{pv}}$ and $\mathbf{\Delta R_{np}}$ or $\mathbf{\alpha_D J}$ and $\mathbf{\Delta R_{np}}$
- **Fitting masses and radii** do not give enough **information** on $\mathbf{A_{pv}}$**,** $\mathbf{\alpha_D}$ and $\mathbf{\Delta R_{np}}$ (additional reason for the strong correlation)
- **Extending** the **fitting protocol** to include $\mathbf{A_{pv}}$ and $\mathbf{\alpha_D}$ do **not change** the **correlations** above since **experimental errors** on these observables **are** still **large → model predictions** remain **biased** by **masses** and **radii**
- **Current EDFs** are able to overlap within **~1σ** all experimental **data** (preliminary: with the exception of A_{pv} in ^{48}Ca that is about 1.5σ away)
- More **accurate measurements** on $\mathbf{A_{pv}}$ could point to **model deficiencies,** while **EDFs** seem to **accommodate** better $\mathbf{\alpha_D}$ (if also looking to other nuclei)
- **Ideally:** measure $\mathbf{A_{pv}}$ at **different kinematics** or $\mathbf{A_{pv}}$ on **different nuclei** with same (or better) accuracy → better **constraints** to **models**

Collaborators

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